

13 Limieten en asymptoten

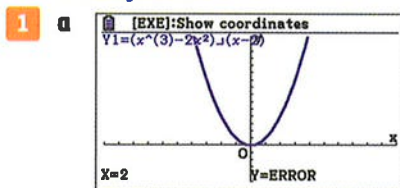
Voorkennis Limiet en afgeleide

Bladzijde 9

- 1 a $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{a(x+h) - ax}{h} = \lim_{h \rightarrow 0} \frac{ax + ah - ax}{h} = \lim_{h \rightarrow 0} \frac{ah}{h} = \lim_{h \rightarrow 0} a = a$
- b $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{a(x+h)^2 - ax^2}{h} = \lim_{h \rightarrow 0} \frac{a(x^2 + 2xh + h^2) - ax^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{ax^2 + 2axh + ah^2 - ax^2}{h} = \lim_{h \rightarrow 0} \frac{2axh + ah^2}{h} = \lim_{h \rightarrow 0} \frac{h(2ax + ah)}{h} = \lim_{h \rightarrow 0} (2ax + ah) = 2ax$
- 2 a $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{g^{x+h} - g^x}{h} = \lim_{h \rightarrow 0} \frac{g^x \cdot g^h - g^x}{h} = \lim_{h \rightarrow 0} \frac{g^x \cdot (g^h - 1)}{h} = \lim_{h \rightarrow 0} \frac{g^h - 1}{h} \cdot g^x$
- b Uit a volgt $f(x) = e^x$ geeft $f'(x) = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} \cdot e^x$.
 $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1$, dus $f'(x) = 1 \cdot e^x = e^x$.

13.1 Limieten en perforaties

Bladzijde 10



- b $f(1) = 1, f(1,9) = 3,61, f(1,99) = 3,9601, f(2,01) = 4,0401$
- c $f(2) = \frac{2^3 - 2 \cdot 2^2}{2 - 2} = \frac{0}{0}$ en dat is onbepaald.

Bladzijde 11

- 2 a $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x - 2} = \frac{(-3)^2 - 3 - 6}{-3 - 2} = \frac{0}{-5} = 0$
- b $\lim_{x \rightarrow 0} \frac{x^2 + 7x + 12}{x^2 + 5x + 6} = \frac{0 + 0 + 12}{0 + 0 + 6} = \frac{12}{6} = 2$
- c $\lim_{x \rightarrow -2} \frac{x^2 + 7x + 12}{x^2 + 5x + 6} = \lim_{x \rightarrow -2} \frac{x + 4}{x + 2} = \frac{2}{0}$ en dat is onbepaald, dus $\lim_{x \rightarrow -2} \frac{x^2 + 7x + 12}{x^2 + 5x + 6}$ bestaat niet.

Bladzijde 12

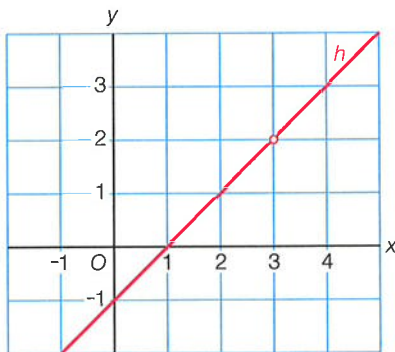
- 3 a $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x \rightarrow 3} \frac{(x+1)(x-3)}{x-3} = \lim_{x \rightarrow 3} (x+1) = 4$
- b $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x + 3} = \lim_{x \rightarrow -3} \frac{(x-1)(x+3)}{x+3} = \lim_{x \rightarrow -3} (x-1) = -4$
- c $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 + x - 2} = \lim_{x \rightarrow 1} \frac{(x-1)(x-2)}{(x-1)(x+2)} = \lim_{x \rightarrow 1} \frac{x-2}{x+2} = \frac{-1}{3} = -\frac{1}{3}$
- d $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^2 + 3x + 2} = \lim_{x \rightarrow -1} \frac{(x+1)(x-2)}{(x+1)(x+2)} = \lim_{x \rightarrow -1} \frac{x-2}{x+2} = \frac{-3}{1} = -3$

- 4** a $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x - 2} = \lim_{x \rightarrow 2} \frac{(x-2)(x-3)}{x-2} = \lim_{x \rightarrow 2} (x-3) = -1$
 b $\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} = \lim_{x \rightarrow 4} \frac{(x-4)(x+4)}{x-4} = \lim_{x \rightarrow 4} (x+4) = 8$
 c $\lim_{x \rightarrow 4} \frac{x^2 - 8x + 16}{x^2 - 16} = \lim_{x \rightarrow 4} \frac{(x-4)^2}{(x-4)(x+4)} = \lim_{x \rightarrow 4} \frac{x-4}{x+4} = \frac{0}{8} = 0$
 d $\lim_{x \rightarrow 0} \frac{2x^2}{3x^2} = \lim_{x \rightarrow 0} \frac{2}{3} = \frac{2}{3}$
- 5** a $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5} = \lim_{x \rightarrow 5} \frac{(x-5)(x+5)}{x-5} = \lim_{x \rightarrow 5} (x+5) = 10$
 b $\lim_{x \rightarrow 0} \frac{x^2 + 2x}{x^2 + x} = \lim_{x \rightarrow 0} \frac{x(x+2)}{x(x+1)} = \lim_{x \rightarrow 0} \frac{x+2}{x+1} = \frac{2}{1} = 2$
 c $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - x - 2} = \frac{1 - 3 + 2}{1 - 1 - 2} = \frac{0}{-2} = 0$
 d $\lim_{x \rightarrow 2} \frac{x^3 + x^2 - 6x}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{x(x^2 + x - 6)}{x^2 - 5x + 6} = \lim_{x \rightarrow 2} \frac{x(x-2)(x+3)}{(x-2)(x-3)} = \lim_{x \rightarrow 2} \frac{x(x+3)}{x-3} = \frac{2 \cdot 5}{-1} = -10$
- 6** a $\lim_{x \rightarrow 6} \frac{x^2 - 36}{x - 6} = \lim_{x \rightarrow 6} \frac{(x-6)(x+6)}{x-6} = \lim_{x \rightarrow 6} (x+6) = 12$
 b $\lim_{x \rightarrow -4} \frac{x^2 + 5x + 4}{x^2 + 3x - 4} = \lim_{x \rightarrow -4} \frac{(x+1)(x+4)}{(x-1)(x+4)} = \lim_{x \rightarrow -4} \frac{x+1}{x-1} = \frac{-3}{-5} = \frac{3}{5}$
 c $\lim_{x \rightarrow -5} \frac{x^2 - 6x + 5}{x^2 - 4x - 5} = \frac{25 + 30 + 5}{25 + 20 - 5} = \frac{60}{40} = 1\frac{1}{2}$
 d $\lim_{x \rightarrow \frac{1}{2}} \frac{4x - 2}{4x^2 - 1} = \lim_{x \rightarrow \frac{1}{2}} \frac{2(2x - 1)}{(2x - 1)(2x + 1)} = \lim_{x \rightarrow \frac{1}{2}} \frac{2}{2x + 1} = \frac{2}{2} = 1$
- 7** a $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 - 7x + 12}{x^2 + x - 12} = \lim_{x \rightarrow 3} \frac{(x-3)(x-4)}{(x-3)(x+4)} = \lim_{x \rightarrow 3} \frac{x-4}{x+4} = \frac{-1}{7} = -\frac{1}{7}$
 De continuummakende waarde van f voor $x = 3$ is $-\frac{1}{7}$.
 b $\lim_{x \rightarrow \frac{1}{4}} g(x) = \lim_{x \rightarrow \frac{1}{4}} \frac{16x^2 - 1}{16x - 4} = \lim_{x \rightarrow \frac{1}{4}} \frac{(4x-1)(4x+1)}{4(4x-1)} = \lim_{x \rightarrow \frac{1}{4}} \frac{4x+1}{4} = \frac{2}{4} = \frac{1}{2}$
 De continuummakende waarde van g voor $x = \frac{1}{4}$ is $\frac{1}{2}$.
- 8** a $\lim_{x \rightarrow \sqrt{2}} \frac{x^2 - 2}{x^2 + x\sqrt{2} - 4} = \lim_{x \rightarrow \sqrt{2}} \frac{(x-\sqrt{2})(x+\sqrt{2})}{(x-\sqrt{2})(x+2\sqrt{2})} = \lim_{x \rightarrow \sqrt{2}} \frac{x+\sqrt{2}}{x+2\sqrt{2}} = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$
 b $\lim_{x \rightarrow \frac{1}{2}} \frac{2x^2 + 7x - 4}{2x^2 + 15x - 8} = \lim_{x \rightarrow \frac{1}{2}} \frac{(2x-1)(x+4)}{(2x-1)(x+8)} = \lim_{x \rightarrow \frac{1}{2}} \frac{x+4}{x+8} = \frac{4\frac{1}{2}}{8\frac{1}{2}} = \frac{9}{17}$

Bladzijde 13

9 $h(x) = \frac{x^2 - 4x + 3}{x - 3} = \frac{(x-1)(x-3)}{x-3} = x - 1$ mits $x \neq 3$

$\lim_{x \rightarrow 3} h(x) = 2$, dus de coördinaten van dit 'gaatje' zijn $(3, 2)$.



Bladzijde 15

10 a $f_a(x) = \frac{x^2 + x - 6}{x + a} = \frac{(x-2)(x+3)}{x+a}$

Er is een perforatie als $a = -2$ en als $a = 3$.

$$\lim_{x \rightarrow 2} f_{-2}(x) = \lim_{x \rightarrow 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x \rightarrow 2} (x+3) = 5$$

Voor $a = -2$ is de perforatie $(2, 5)$.

$$\lim_{x \rightarrow 3} f_3(x) = \lim_{x \rightarrow 3} \frac{(x-2)(x+3)}{x+3} = \lim_{x \rightarrow 3} (x-2) = -5$$

Voor $a = 3$ is de perforatie $(-3, -5)$.

b $f_a(x) = \frac{x^2 + x - 20}{x + a} = \frac{(x-4)(x+5)}{x+a}$

Er is een perforatie als $a = -4$ en als $a = 5$.

$$\lim_{x \rightarrow 4} f_{-4}(x) = \lim_{x \rightarrow 4} \frac{(x-4)(x+5)}{x-4} = \lim_{x \rightarrow 4} (x+5) = 9$$

Voor $a = -4$ is de perforatie $(4, 9)$.

$$\lim_{x \rightarrow 5} f_5(x) = \lim_{x \rightarrow 5} \frac{(x-4)(x+5)}{x+5} = \lim_{x \rightarrow 5} (x-4) = -9$$

Voor $a = 5$ is de perforatie $(-5, -9)$.

c $f_a(x) = \frac{x^2 - 9}{x + a} = \frac{(x-3)(x+3)}{x+a}$

Er is een perforatie als $a = -3$ en als $a = 3$.

$$\lim_{x \rightarrow 3} f_{-3}(x) = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x \rightarrow 3} (x+3) = 6$$

Voor $a = -3$ is de perforatie $(3, 6)$.

$$\lim_{x \rightarrow 3} f_3(x) = \lim_{x \rightarrow 3} \frac{(x-3)(x+3)}{x+3} = \lim_{x \rightarrow 3} (x-3) = -6$$

Voor $a = 3$ is de perforatie $(-3, -6)$.

11 a $f_a(x) = \frac{x^2 - 4\frac{1}{2}x + 2}{x + a} = \frac{(x-4)(x-\frac{1}{2})}{x+a}$

Er is een perforatie als $a = -4$ en als $a = -\frac{1}{2}$.

$$\lim_{x \rightarrow 4} f_{-4}(x) = \lim_{x \rightarrow 4} \frac{(x-4)(x-\frac{1}{2})}{x-4} = \lim_{x \rightarrow 4} (x-\frac{1}{2}) = 3\frac{1}{2}$$

Voor $a = -4$ is de perforatie $(4, 3\frac{1}{2})$.

$$\lim_{x \rightarrow \frac{1}{2}} f_{-\frac{1}{2}}(x) = \lim_{x \rightarrow \frac{1}{2}} \frac{(x-4)(x-\frac{1}{2})}{x-\frac{1}{2}} = \lim_{x \rightarrow \frac{1}{2}} (x-4) = -3\frac{1}{2}$$

Voor $a = -\frac{1}{2}$ is de perforatie $(\frac{1}{2}, -3\frac{1}{2})$.

b $f_a(x) = \frac{x^2 - 3x\sqrt{2} + 4}{x + a} = \frac{(x-\sqrt{2})(x-2\sqrt{2})}{x+a}$

Er is een perforatie als $a = -\sqrt{2}$ en als $a = -2\sqrt{2}$.

$$\lim_{x \rightarrow \sqrt{2}} f_{-\sqrt{2}}(x) = \lim_{x \rightarrow \sqrt{2}} \frac{(x-\sqrt{2})(x-2\sqrt{2})}{x-\sqrt{2}} = \lim_{x \rightarrow \sqrt{2}} (x-2\sqrt{2}) = -\sqrt{2}$$

Voor $a = -\sqrt{2}$ is de perforatie $(\sqrt{2}, -\sqrt{2})$.

$$\lim_{x \rightarrow 2\sqrt{2}} f_{-2\sqrt{2}}(x) = \lim_{x \rightarrow 2\sqrt{2}} \frac{(x-\sqrt{2})(x-2\sqrt{2})}{x-2\sqrt{2}} = \lim_{x \rightarrow 2\sqrt{2}} (x-\sqrt{2}) = \sqrt{2}$$

Voor $a = -2\sqrt{2}$ is de perforatie $(2\sqrt{2}, \sqrt{2})$.

$$c \quad f_a(x) = \frac{2x^2 - x - 3}{2x + a} = \frac{(x+1)(2x-3)}{2x+a} = \frac{\frac{1}{2}(2x+2)(2x-3)}{2x+a}$$

Er is een perforatie als $a = 2$ en als $a = -3$.

$$\lim_{x \rightarrow -1} f_2(x) = \lim_{x \rightarrow -1} \frac{\frac{1}{2}(2x+2)(2x-3)}{2x+2} = \lim_{x \rightarrow -1} \frac{1}{2}(2x-3) = \frac{1}{2}(-2-3) = -2\frac{1}{2}$$

Voor $a = 2$ is de perforatie $(-1, -2\frac{1}{2})$.

$$\lim_{x \rightarrow 1\frac{1}{2}} f_{-3}(x) = \lim_{x \rightarrow 1\frac{1}{2}} \frac{\frac{1}{2}(2x+2)(2x-3)}{2x-3} = \lim_{x \rightarrow 1\frac{1}{2}} \frac{1}{2}(2x+2) = \frac{1}{2}(3+2) = 2\frac{1}{2}$$

Voor $a = -3$ is de perforatie $(1\frac{1}{2}, 2\frac{1}{2})$.

$$12 \quad a \quad f_a(x) = \frac{9x^2 + 6x - 8}{3x + a} = \frac{(3x-2)(3x+4)}{3x+a}$$

Er is een perforatie als $a = -2$ en als $a = 4$.

$$\lim_{x \rightarrow \frac{2}{3}} f_{-2}(x) = \lim_{x \rightarrow \frac{2}{3}} \frac{(3x-2)(3x+4)}{3x-2} = \lim_{x \rightarrow \frac{2}{3}} (3x+4) = 2+4=6$$

Voor $a = -2$ is de perforatie $(\frac{2}{3}, 6)$.

$$\lim_{x \rightarrow -1\frac{1}{3}} f_4(x) = \lim_{x \rightarrow -1\frac{1}{3}} \frac{(3x-2)(3x+4)}{3x+4} = \lim_{x \rightarrow -1\frac{1}{3}} (3x-2) = -4-2=-6$$

Voor $a = 4$ is de perforatie $(-1\frac{1}{3}, -6)$.

$$b \quad f_a(x) = \frac{4x^2 - 4x - 15}{2x + a} = \frac{(2x+3)(2x-5)}{2x+a}$$

Er is een perforatie als $a = 3$ en als $a = -5$.

$$\lim_{x \rightarrow -1\frac{1}{2}} f_3(x) = \lim_{x \rightarrow -1\frac{1}{2}} \frac{(2x+3)(2x-5)}{2x+3} = \lim_{x \rightarrow -1\frac{1}{2}} (2x-5) = -3-5=-8$$

Voor $a = 3$ is de perforatie $(-1\frac{1}{2}, -8)$.

$$\lim_{x \rightarrow 2\frac{1}{2}} f_{-5}(x) = \lim_{x \rightarrow 2\frac{1}{2}} \frac{(2x+3)(2x-5)}{2x-5} = \lim_{x \rightarrow 2\frac{1}{2}} (2x+3) = 5+3=8$$

Voor $a = -5$ is de perforatie $(2\frac{1}{2}, 8)$.

$$c \quad f_a(x) = \frac{2x^2 - 11x + 15}{2x + a} = \frac{(x-3)(2x-5)}{2x+a} = \frac{\frac{1}{2}(2x-6)(2x-5)}{2x+a}$$

Er is een perforatie als $a = -6$ en als $a = -5$.

$$\lim_{x \rightarrow 3} f_{-6}(x) = \lim_{x \rightarrow 3} \frac{\frac{1}{2}(2x-6)(2x-5)}{2x-6} = \lim_{x \rightarrow 3} (\frac{1}{2}(2x-5)) = \frac{1}{2}(6-5) = \frac{1}{2}$$

Voor $a = -6$ is de perforatie $(3, \frac{1}{2})$.

$$\lim_{x \rightarrow 2\frac{1}{2}} f_{-5}(x) = \lim_{x \rightarrow 2\frac{1}{2}} \frac{\frac{1}{2}(2x-6)(2x-5)}{2x-5} = \lim_{x \rightarrow 2\frac{1}{2}} (\frac{1}{2}(2x-6)) = \frac{1}{2}(5-6) = -\frac{1}{2}$$

Voor $a = -5$ is de perforatie $(2\frac{1}{2}, -\frac{1}{2})$.

- 13 Voor een perforatie mag de functiewaarde niet bestaan. Dat is alleen het geval als de noemer gelijk is aan nul. Maar als de teller voor een nulpunt van de noemer ongelijk is aan nul, dan bestaat de limiet van $f(x)$ niet. De enige mogelijkheden voor een perforatie is er dus als de teller hetzelfde nulpunt heeft als de noemer. Je moet nog wel controleren of de limiet van $f(x)$ bestaat.

$$14 \quad f_a(x) = \frac{x^2 - 4ax + 3a^2}{x - a} = \frac{(x-a)(x-3a)}{x-a}$$

$$\lim_{x \rightarrow a} f_a(x) = \lim_{x \rightarrow a} \frac{(x-a)(x-3a)}{x-a} = \lim_{x \rightarrow a} (x-3a) = -2a$$

De perforatie is $(a, -2a)$.

$(a, -2a)$ op de lijn $y = x - 3$ geeft $a - 3 = -2a$

$$3a = 3$$

$$a = 1$$

Bladzijde 16

$$15 \quad a \quad f_a(x) = \frac{2x^2 + x - 10}{2x + a} = \frac{(2x + 5)(x - 2)}{2x + a} = \frac{\frac{1}{2}(2x + 5)(2x - 4)}{2x + a}$$

Er is een perforatie als $a = 5$ en als $a = -4$.

$$\lim_{x \rightarrow -2\frac{1}{2}} f_5(x) = \lim_{x \rightarrow -2\frac{1}{2}} \frac{\frac{1}{2}(2x + 5)(2x - 4)}{2x + 5} = \lim_{x \rightarrow -2\frac{1}{2}} (\frac{1}{2}(2x - 4)) = \frac{1}{2}(-5 - 4) = -4\frac{1}{2}$$

Dus voor $a = 5$ is de perforatie $(-2\frac{1}{2}, -4\frac{1}{2})$.

$$\lim_{x \rightarrow 2} f_{-4}(x) = \lim_{x \rightarrow 2} \frac{\frac{1}{2}(2x + 5)(2x - 4)}{2x - 4} = \lim_{x \rightarrow 2} (\frac{1}{2}(2x + 5)) = \frac{1}{2}(4 + 5) = 4\frac{1}{2}$$

Dus voor $a = -4$ is de perforatie $(2, 4\frac{1}{2})$.

$$b \quad f_a(x) = \frac{2x^2 + x - 10}{2x + a} \text{ geeft}$$

$$f'_a(x) = \frac{(2x + a) \cdot (4x + 1) - (2x^2 + x - 10) \cdot 2}{(2x + a)^2} = \frac{8x^2 + 2x + 4ax + a - 4x^2 - 2x + 20}{(2x + a)^2} = \frac{4x^2 + 4ax + a + 20}{(2x + a)^2}$$

$$f'_a(-1) = 0 \text{ geeft } \frac{4 - 4a + a + 20}{(-2 + a)^2} = 0$$

$$4 - 4a + a + 20 = 0$$

$$-3a = -24$$

$$a = 8$$

$$f_8(x) = \frac{2x^2 + x - 10}{2x + 8} \text{ en } f'_8(x) = \frac{4x^2 + 32x + 28}{(2x + 8)^2}$$

$$f'_8(x) = 0 \text{ geeft } 4x^2 + 32x + 28 = 0$$

$$x^2 + 8x + 7 = 0$$

$$(x + 1)(x + 7) = 0$$

$$x = -1 \vee x = -7$$

$$f_8(-7) = \frac{2 \cdot 49 - 7 - 10}{-14 + 8} = \frac{81}{-6} = -13\frac{1}{2}$$

De coördinaten van de andere top zijn $(-7, -13\frac{1}{2})$.

$$16 \quad f_a(x) = \frac{4x^2 - 14ax + 6a^2}{2x - a} = \frac{(2x - a)(2x - 6a)}{2x - a}$$

Voor elke waarde van a is er een perforatie als $x = \frac{1}{2}a$.

$$\lim_{x \rightarrow \frac{1}{2}a} f_a(x) = \lim_{x \rightarrow \frac{1}{2}a} \frac{(2x - a)(2x - 6a)}{2x - a} = \lim_{x \rightarrow \frac{1}{2}a} (2x - 6a) = a - 6a = -5a$$

De perforatie is $(\frac{1}{2}a, -5a)$.

$$(\frac{1}{2}a, -5a) \text{ op } y = -2x^2 + 12 \text{ geeft } -2 \cdot \frac{1}{4}a^2 + 12 = -5a$$

$$-\frac{1}{2}a^2 + 5a + 12 = 0$$

$$a^2 - 10a - 24 = 0$$

$$(a + 2)(a - 12) = 0$$

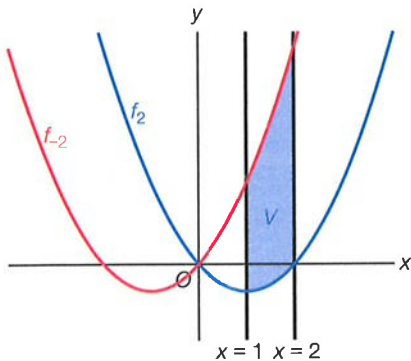
$$a = -2 \vee a = 12$$

$$17 \quad f_a(x) = \frac{x^3 - 4x}{x + a} = \frac{x(x^2 - 4)}{x + a} = \frac{x(x-2)(x+2)}{x + a}$$

Er is een perforatie als $a = 0$ (vold. niet), $a = -2$ en $a = 2$.

$$f_{-2}(x) = \frac{x(x-2)(x+2)}{x-2} = x(x+2) = x^2 + 2x \text{ mits } x \neq 2$$

$$f_2(x) = \frac{x(x-2)(x+2)}{x+2} = x(x-2) = x^2 - 2x \text{ mits } x \neq -2$$

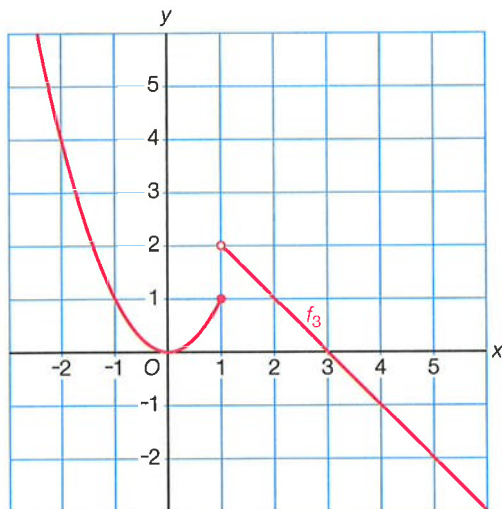


$$O(V) = \int_1^2 (f_{-2}(x) - f_2(x)) dx = \int_1^2 (x^2 + 2x - (x^2 - 2x)) dx = \int_1^2 4x dx = [2x^2]_1^2 = 2 \cdot 4 - 2 \cdot 1 = 6$$

13.2 Sprongen en knikken in grafieken

Bladzijde 18

$$18 \quad a \quad f_3(x) = \begin{cases} x^2 & \text{voor } x \leq 1 \\ -x + 3 & \text{voor } x > 1 \end{cases}$$



- b Nee, de grafiek van f_3 is geen ononderbroken kromme.
c Voor $p = 2$ is de grafiek van f_p een ononderbroken kromme.

Bladzijde 19

$$19 \quad a \quad \lim_{x \uparrow 1} f_p(x) = \lim_{x \uparrow 1} 2^{x-p} = 2^{1-p}$$

$$\lim_{x \downarrow 1} f_p(x) = \lim_{x \downarrow 1} (x^2 + 7) = 1 + 7 = 8$$

$$\lim_{x \rightarrow 1} f_p(x) \text{ bestaat als } 2^{1-p} = 8$$

$$1 - p = 3$$

$$p = -2$$

b $\lim_{x \uparrow 1} f_p(x) = \lim_{x \uparrow 1} e^{x+p} = e^{1+p}$
 $\lim_{x \downarrow 1} f_p(x) = \lim_{x \downarrow 1} (x+3) = 1+3 = 4$
 $\lim_{x \rightarrow 1} f_p(x)$ bestaat als $e^{1+p} = 4$
 $1+p = \ln(4)$
 $p = -1 + \ln(4)$

c $\lim_{x \uparrow 1} f_p(x) = \lim_{x \uparrow 1} \ln(x+p) = \ln(1+p)$
 $\lim_{x \downarrow 1} f_p(x) = \lim_{x \downarrow 1} (x^2+1) = 1+1 = 2$
 $\lim_{x \rightarrow 1} f_p(x)$ bestaat als $\ln(1+p) = 2$
 $1+p = e^2$
 $p = -1 + e^2$

d $\lim_{x \uparrow 1} f_p(x) = \lim_{x \uparrow 1} |px-2| = |p-2|$
 $\lim_{x \downarrow 1} f_p(x) = \lim_{x \downarrow 1} (x^2+3x) = 1+3 = 4$
 $\lim_{x \rightarrow 1} f_p(x)$ bestaat als $|p-2| = 4$
 $p-2 = 4 \vee p-2 = -4$
 $p = 6 \vee p = -2$

Bladzijde 20

20 $\lim_{x \uparrow 2} f_{p,q}(x) = \lim_{x \uparrow 2} (x^2+p) = 4+p$
 $\lim_{x \downarrow 2} f_{p,q}(x) = \lim_{x \downarrow 2} (3x+q) = 6+q$
 $\lim_{x \uparrow 4} f_{p,q}(x) = \lim_{x \uparrow 4} (3x+q) = 12+q$
 $\lim_{x \downarrow 4} f_{p,q}(x) = \lim_{x \downarrow 4} (-x^2-px+11) = -16-4p+11 = -4p-5$
 $\lim_{x \rightarrow 2} f_{p,q}(x)$ bestaat als $4+p = 6+q$
 $p-q = 2$
 $\lim_{x \rightarrow 4} f_{p,q}(x)$ bestaat als $12+q = -4p-5$
 $4p+q = -17$

$$\begin{cases} p-q=2 \\ 4p+q=-17 \end{cases} \rightarrow \begin{matrix} 5p = -15 \\ p = -3 \end{matrix}$$

$$\begin{cases} p-q=2 \\ p=-3 \end{cases} \rightarrow \begin{matrix} -3-q=2 \\ q=-5 \end{matrix}$$

Dus voor $p = -3$ en $q = -5$.

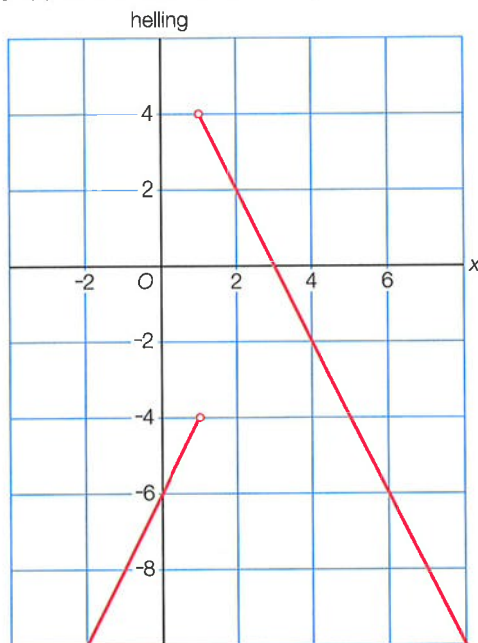
21 $\lim_{x \uparrow \frac{1}{4}} f_{p,q}(x) = \lim_{x \uparrow \frac{1}{4}} 4 \sin(\pi x) = 4 \sin(\frac{1}{4}\pi p)$
 $\lim_{x \downarrow \frac{1}{4}} f_{p,q}(x) = \lim_{x \downarrow \frac{1}{4}} (8x^2+qx+2) = \frac{1}{2} + \frac{1}{4}q + 2 = \frac{1}{4}q + 2\frac{1}{2}$
 $\lim_{x \uparrow 3} f_{p,q}(x) = \lim_{x \uparrow 3} (8x^2+qx+2) = 72+3q+2 = 3q+74$
 $\lim_{x \downarrow 3} f_{p,q}(x) = \lim_{x \downarrow 3} (x^4-4x-1) = 81-12-1 = 68$
 $\lim_{x \rightarrow \frac{1}{4}} f_{p,q}(x)$ bestaat als $3q+74 = 68$
 $3q = -6$
 $q = -2$
 $\lim_{x \rightarrow \frac{1}{4}} f_{p,q}(x)$ bestaat als $4 \sin(\frac{1}{4}\pi p) = \frac{1}{4}q + 2\frac{1}{2}$
 $q = -2$ geeft $4 \sin(\frac{1}{4}\pi p) = -\frac{1}{2} + 2\frac{1}{2}$
 $4 \sin(\frac{1}{4}\pi p) = 2$
 $\sin(\frac{1}{4}\pi p) = \frac{1}{2}$
 $\frac{1}{4}\pi p = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{4}\pi p = \frac{5}{6}\pi + k \cdot 2\pi$
 $p = \frac{2}{3} + k \cdot 8 \vee p = 3\frac{1}{3} + k \cdot 8$
 $0 < p < 5$ geeft $p = \frac{2}{3} \vee p = 3\frac{1}{3}$
Dus voor $(p = \frac{2}{3} \vee p = 3\frac{1}{3}) \wedge q = -2$.

- 22 a Voor $x < 2$ is $f'(x) = x \cdot (-x + 2) = -x^2 + 2x$.
 b Voor $x \geq 2$ is $f'(x) = x \cdot (x - 2) = x^2 - 2x$.
 c Voor $x > 2$ is $f'(x) = 2x - 2$.
 d $\lim_{x \uparrow 2} f'(x) = \lim_{x \uparrow 2} (-2x + 2) = -4 + 2 = -2$
 $\lim_{x \downarrow 2} f'(x) = \lim_{x \downarrow 2} (2x - 2) = 4 - 2 = 2$
 e $\lim_{x \uparrow 2} f'(x) \neq \lim_{x \downarrow 2} f'(x)$, dus $\lim_{x \rightarrow 2} f'(x)$ bestaat niet.

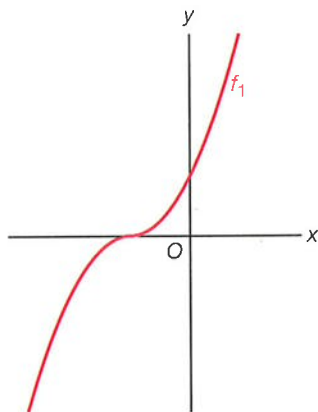
Bladzijde 21

- 23 a De notatie $f'(x) = \begin{cases} -2x + 6 & \text{voor } x > 1 \\ 2x - 6 & \text{voor } x < 1 \end{cases}$ is juist, want $\lim_{x \rightarrow 1} f'(x)$ bestaat niet oftewel $f'(x)$ bestaat niet voor $x = 1$.

b



- 24 a $f_1(x) = \begin{cases} x^2 + 2x + 1 & \text{voor } x \geq -1 \\ -x^2 - 2x - 1 & \text{voor } x < -1 \end{cases}$



- b $f_p(3) = 4$ geeft $(3 + 1) \cdot |3 + p| = 4$
 $|3 + p| = 1$
 $3 + p = 1 \vee 3 + p = -1$
 $p = -2 \vee p = -4$

Bladzijde 22

25 a $f_1(x) = \begin{cases} (2x+1)(x-1) = 2x^2 - x - 1 & \text{voor } x \geq 1 \\ (2x+1)(-x+1) = -2x^2 + x + 1 & \text{voor } x < 1 \end{cases}$

Het knikpunt is $(1, 0)$.

$$\lim_{x \uparrow 1} f'(x) = \lim_{x \uparrow 1} (-4x + 1) = -4 + 1 = -3$$

Dus $k: y = -3x + 3$.

$$\lim_{x \downarrow 1} f'(x) = \lim_{x \downarrow 1} (4x - 1) = 4 - 1 = 3$$

Dus $l: y = 3x - 3$.

b $f_p(-1) = -3$ geeft $(-2+1) \cdot |-1-p| = -3$
 $|-1-p| = 3$
 $-1-p = 3 \vee -1-p = -3$
 $p = -4 \vee p = 2$

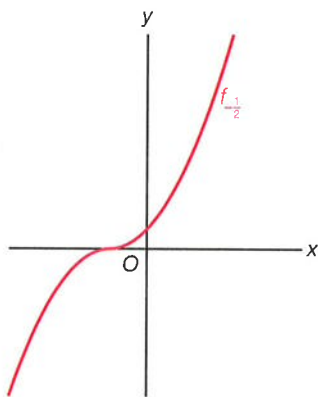
c $f_p(x) = \begin{cases} (2x+1)(x-p) = 2x^2 - 2px + x - p & \text{voor } x \geq p \\ (2x+1)(-x+p) = -2x^2 + 2px - x + p & \text{voor } x < p \end{cases}$

$$\lim_{x \uparrow p} f'_p(x) = \lim_{x \uparrow p} (-4x + 2p - 1) = -4p + 2p - 1 = -2p - 1$$

$$\lim_{x \downarrow p} f'_p(x) = \lim_{x \downarrow p} (4x - 2p + 1) = 4p - 2p + 1 = 2p + 1$$

Geen knik als $\lim_{x \uparrow p} f'_p(x) = \lim_{x \downarrow p} f'_p(x)$, dus als $-2p - 1 = 2p + 1$
 $-4p = 2$
 $p = -\frac{1}{2}$

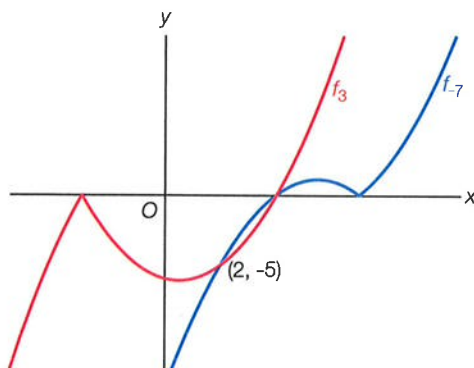
$$f_{-\frac{1}{2}}(x) = \begin{cases} 2x^2 + 2x + \frac{1}{2} & \text{voor } x \geq -\frac{1}{2} \\ -2x^2 - 2x - \frac{1}{2} & \text{voor } x < -\frac{1}{2} \end{cases}$$



26 a $f_p(2) = -5$ geeft $(1-2) \cdot |2+p| = -5$
 $|2+p| = 5$
 $2+p = 5 \vee 2+p = -5$
 $p = 3 \vee p = -7$

$$f_3(x) = \begin{cases} \frac{1}{2}x^2 - \frac{1}{2}x - 6 & \text{voor } x \geq -3 \\ -\frac{1}{2}x^2 + \frac{1}{2}x + 6 & \text{voor } x < -3 \end{cases}$$

$$f_{-7}(x) = \begin{cases} \frac{1}{2}x^2 - 5\frac{1}{2}x + 14 & \text{voor } x \geq 7 \\ -\frac{1}{2}x^2 + 5\frac{1}{2}x - 14 & \text{voor } x < 7 \end{cases}$$



b $f_p(x) = \begin{cases} (\frac{1}{2}x - 2)(x + p) = \frac{1}{2}x^2 + \frac{1}{2}px - 2x - 2p & \text{voor } x \geq -p \\ (\frac{1}{2}x - 2)(-x - p) = -\frac{1}{2}x^2 - \frac{1}{2}px + 2x + 2p & \text{voor } x < -p \end{cases}$

$$\lim_{x \uparrow -p} f_p'(x) = \lim_{x \uparrow -p} (-x - \frac{1}{2}p + 2) = p - \frac{1}{2}p + 2 = \frac{1}{2}p + 2$$

$$\lim_{x \downarrow -p} f_p'(x) = \lim_{x \downarrow -p} (x + \frac{1}{2}p - 2) = -p + \frac{1}{2}p - 2 = -\frac{1}{2}p - 2$$

Geen knik als $\lim_{x \uparrow -p} f_p'(x) = \lim_{x \downarrow -p} f_p'(x)$, dus als $\frac{1}{2}p + 2 = -\frac{1}{2}p - 2$
 $p = -4$

c $f_4(x) = \begin{cases} \frac{1}{2}x^2 - 8 & \text{voor } x \geq -4 \\ -\frac{1}{2}x^2 + 8 & \text{voor } x < -4 \end{cases}$

$$\lim_{x \uparrow -4} f_4'(x) = \lim_{x \uparrow -4} -x = 4, \text{ dus } rc_k = 4.$$

$$rc_k = \tan(\alpha) = 4, \text{ dus } \alpha = 75,9...^\circ.$$

$$\lim_{x \downarrow -4} f_4'(x) = \lim_{x \downarrow -4} x = -4, \text{ dus } rc_l = -4.$$

$$rc_l = \tan(\beta) = -4, \text{ dus } \beta = -75,9...^\circ.$$

$$\alpha - \beta = 75,9...^\circ - (-75,9...^\circ) \approx 152^\circ$$

Dus $\angle(k, l) \approx 180^\circ - 152^\circ = 28^\circ$.

d Zie b.

$$rc_k = \frac{1}{2}p + 2 \text{ en } rc_l = -\frac{1}{2}p - 2$$

$$k \perp l, \text{ dus } rc_k \cdot rc_l = -1$$

$$(\frac{1}{2}p + 2)(-\frac{1}{2}p - 2) = -1$$

$$(\frac{1}{2}p + 2)(\frac{1}{2}p + 2) = 1$$

$$\frac{1}{4}p^2 + 2p + 4 = 1$$

$$\frac{1}{4}p^2 + 2p + 3 = 0$$

$$p^2 + 8p + 12 = 0$$

$$(p + 2)(p + 6) = 0$$

$$p = -2 \vee p = -6$$

27 a $f_p(x) = \begin{cases} (x - p) \ln(x) & \text{voor } x \geq p \\ (-x + p) \ln(x) & \text{voor } x < p \end{cases}$

$$[(x - p) \ln(x)]' = 1 \cdot \ln(x) + (x - p) \cdot \frac{1}{x} = \ln(x) + 1 - \frac{p}{x}$$

$$[(-x + p) \ln(x)]' = -1 \cdot \ln(x) + (-x + p) \cdot \frac{1}{x} = -\ln(x) - 1 + \frac{p}{x}$$

$$\lim_{x \uparrow p} f_p'(x) = \lim_{x \uparrow p} \left(-\ln(x) - 1 + \frac{p}{x} \right) = -\ln(p) - 1 + \frac{p}{p} = -\ln(p) - 1 + 1 = -\ln(p)$$

$$\lim_{x \downarrow p} f_p'(x) = \lim_{x \downarrow p} \left(\ln(x) + 1 - \frac{p}{x} \right) = \ln(p) + 1 - \frac{p}{p} = \ln(p) + 1 - 1 = \ln(p)$$

Geen knik als $\lim_{x \uparrow p} f_p'(x) = \lim_{x \downarrow p} f_p'(x)$, dus als $-\ln(p) = \ln(p)$
 $-2 \ln(p) = 0$
 $\ln(p) = 0$
 $p = 1$

b $\lim_{x \uparrow 2} f_2'(x) = \lim_{x \uparrow 2} \left(-\ln(x) - 1 + \frac{2}{x} \right) = -\ln(2), \text{ dus } rc_k = -\ln(2).$

$$rc_k = \tan(\alpha) = -\ln(2), \text{ dus } \alpha = -34,7...^\circ.$$

$$\lim_{x \downarrow 2} f_2'(x) = \lim_{x \downarrow 2} \left(\ln(x) + 1 - \frac{2}{x} \right) = \ln(2), \text{ dus } rc_l = \ln(2).$$

$$rc_l = \tan(\beta) = \ln(2), \text{ dus } \beta = 34,7...^\circ.$$

$$\beta - \alpha = 34,7...^\circ - (-34,7...^\circ) \approx 69^\circ$$

Dus $\angle(k, l) \approx 69^\circ$.

c Zie a.

$$rc_k = -\ln(p) \text{ en } rc_l = \ln(p)$$

$$k \perp l, \text{ dus } rc_k \cdot rc_l = -1$$

$$-\ln(p) \cdot \ln(p) = -1$$

$$\ln^2(p) = 1$$

$$\ln(p) = 1 \vee \ln(p) = -1$$

$$p = e \vee p = \frac{1}{e}$$

13.3 Asymptoten bij gebroken functies

Bladzijde 24

- 28 a** $f(1,1) \approx -4$
 $f(1,01) \approx -49$
 $f(1,001) \approx -499$
- b** $\lim_{x \uparrow 1} f(x) = \infty$
- c** $f(10) \approx 1,717$
 $f(100) \approx 1,970$
 $f(1000) \approx 1,997$
 $\lim_{x \rightarrow \infty} f(x) = 2$
- d** $\lim_{x \rightarrow -\infty} f(x) = 2$

Bladzijde 26

- 29 a** $f(x) = 2$ geeft $\frac{2x^2 - 3x}{x^2 - 1} = 2$
 $2x^2 - 3x = 2(x^2 - 1)$
 $2x^2 - 3x = 2x^2 - 2$
 $-3x = -2$
 $x = \frac{2}{3}$

Dus het snijpunt is $(\frac{2}{3}, 2)$.

- b** $g(x) = \frac{x^2}{x} = x$ mits $x \neq 0$

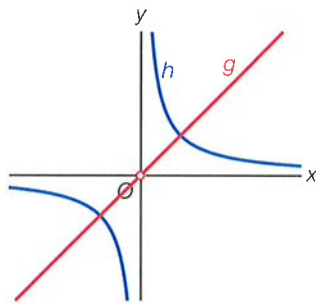
De grafiek van g is de lijn $y = x$ met perforatie $(0, 0)$.

$$h(x) = \frac{x}{x^2} = \frac{1}{x}$$

noemer = 0 geeft $x = 0$, dus de lijn $x = 0$ is verticale asymptoot van de grafiek van h .

$\lim_{x \rightarrow \infty} h(x) = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$, dus de lijn $y = 0$ is horizontale asymptoot van de grafiek van h .

De grafiek van h is de hyperbool $y = \frac{1}{x}$.



- 30 a** $\lim_{x \rightarrow \infty} \frac{3x^2 - x}{2 - x^2} = \lim_{x \rightarrow \infty} \frac{3 - \frac{1}{x}}{\frac{2}{x^2} - 1} = \frac{3 - 0}{0 - 1} = -3$

$$\text{b } \lim_{x \rightarrow -\infty} \frac{5 - x^3}{2x^3} = \lim_{x \rightarrow -\infty} \frac{\frac{5}{x^3} - 1}{2} = \frac{0 - 1}{2} = -\frac{1}{2}$$

$$\text{c } \lim_{x \rightarrow \infty} \frac{6x^2}{2 - x^3} = \lim_{x \rightarrow \infty} \frac{\frac{6}{x}}{\frac{2}{x^3} - 1} = \frac{0}{0 - 1} = 0$$

$$\text{d } \lim_{x \rightarrow \infty} \frac{(2x - 3)^2}{x^2 + 1} = \lim_{x \rightarrow \infty} \frac{4x^2 - 12x + 9}{x^2 + 1} = \lim_{x \rightarrow \infty} \frac{4 - \frac{12}{x} + \frac{9}{x^2}}{1 + \frac{1}{x^2}} = \frac{4 - 0 + 0}{1 + 0} = 4$$

$$31 \quad a \quad \lim_{x \rightarrow \infty} \frac{(3x-1)^2}{(2x+1)^2} = \lim_{x \rightarrow \infty} \frac{9x^2 - 6x + 1}{4x^2 + 4x + 1} = \lim_{x \rightarrow \infty} \frac{9 - \frac{6}{x} + \frac{1}{x^2}}{4 + \frac{4}{x} + \frac{1}{x^2}} = \frac{9-0+0}{4+0+0} = 2\frac{1}{4}$$

$$b \quad \lim_{x \rightarrow -\infty} \frac{|x|+1}{1-|x|} = \lim_{x \rightarrow -\infty} \frac{-x+1}{1+x} = \lim_{x \rightarrow -\infty} \frac{-1 + \frac{1}{x}}{\frac{1}{x} + 1} = \frac{-1+0}{0+1} = -1$$

$$c \quad \lim_{x \rightarrow \infty} \frac{|2-x^2|}{(x+1)^2} = \lim_{x \rightarrow \infty} \frac{-2+x^2}{x^2+2x+1} = \lim_{x \rightarrow \infty} \frac{-\frac{2}{x^2}+1}{1+\frac{2}{x}+\frac{1}{x^2}} = \frac{0+1}{1+0+0} = 1$$

$$d \quad \lim_{x \rightarrow \infty} \frac{x\sqrt{x}}{x^2+1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{\sqrt{x}}}{1+\frac{1}{x^2}} = \frac{0}{1+0} = 0$$

$$32 \quad a \quad \lim_{x \rightarrow \infty} \frac{(4x-1)^2}{x^3+4} = \lim_{x \rightarrow \infty} \frac{16x^2-8x+1}{x^3+4} = \lim_{x \rightarrow \infty} \frac{\frac{16}{x} - \frac{8}{x^2} + \frac{1}{x^3}}{1 + \frac{4}{x^3}} = \frac{0-0+0}{1+0} = 0$$

$$b \quad \lim_{x \rightarrow \infty} \frac{x(2x+1)^2}{x^3+1} = \lim_{x \rightarrow \infty} \frac{x(4x^2+4x+1)}{x^3+1} = \lim_{x \rightarrow \infty} \frac{4x^3+4x^2+x}{x^3+1} = \lim_{x \rightarrow \infty} \frac{4 + \frac{4}{x} + \frac{1}{x^2}}{1 + \frac{1}{x^3}} = \frac{4+0+0}{1+0} = 4$$

$$c \quad \lim_{x \rightarrow \infty} \frac{|x^3-8|}{2x^3+x} = \lim_{x \rightarrow \infty} \frac{-x^3+8}{2x^3+x} = \lim_{x \rightarrow \infty} \frac{-1 + \frac{8}{x^3}}{2 + \frac{1}{x^2}} = \frac{-1+0}{2+0} = -\frac{1}{2}$$

$$d \quad \lim_{x \rightarrow \infty} \frac{|4x^3|-10}{|x^3-1|} = \lim_{x \rightarrow \infty} \frac{-4x^3-10}{-x^3+1} = \lim_{x \rightarrow \infty} \frac{-4 - \frac{10}{x^3}}{-1 + \frac{1}{x^3}} = \frac{-4-0}{-1+0} = 4$$

$$33 \quad a \quad x^2 - x - 6 = 0 \wedge 4x^2 - 1 \neq 0 \\ (x+2)(x-3) = 0 \wedge x^2 \neq \frac{1}{4} \\ x = -2 \vee x = 3$$

De verticale asymptoten zijn de lijnen $x = -2$ en $x = 3$.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{4x^2-1}{x^2-x-6} = \lim_{x \rightarrow \infty} \frac{4 - \frac{1}{x^2}}{1 - \frac{1}{x} - \frac{6}{x^2}} = \frac{4-0}{1-0-0} = 4$$

De horizontale asymptoot is de lijn $y = 4$.

$$b \quad f(x) = 4 \text{ geeft } \frac{4x^2-1}{x^2-x-6} = 4 \\ 4x^2-1 = 4x^2-4x-24 \\ 4x = -23 \\ x = -5\frac{3}{4}$$

Dus $A(-5\frac{3}{4}, 4)$.

$$c \quad f(x) = 4\frac{1}{2} \text{ geeft } \frac{4x^2-1}{x^2-x-6} = \frac{9}{2} \\ 9x^2-9x-54 = 8x^2-2 \\ x^2-9x-52 = 0 \\ (x+4)(x-13) = 0 \\ x = -4 \vee x = 13 \\ f(x) < 4\frac{1}{2} \text{ geeft } x < -4 \vee -2 < x < 3 \vee x > 13$$

Bladzijde 27

34 a $x^3 - 4x = 0 \wedge |x^3| \neq 0$
 $x(x^2 - 4) = 0 \wedge x^3 \neq 0$
 $(x = 0 \vee x^2 = 4) \wedge x \neq 0$
 $x = 2 \vee x = -2$

De verticale asymptoten zijn de lijnen $x = 2$ en $x = -2$.

$$\lim_{x \rightarrow \infty} \frac{|x^3|}{x^3 - 4x} = \lim_{x \rightarrow \infty} \frac{x^3}{x^3 - 4x} = \lim_{x \rightarrow \infty} \frac{1}{1 - \frac{4}{x^2}} = \frac{1}{1 - 0} = 1$$

Voor $x \rightarrow \infty$ is de horizontale asymptoot de lijn $y = 1$.

$$\lim_{x \rightarrow -\infty} \frac{|x^3|}{x^3 - 4x} = \lim_{x \rightarrow -\infty} \frac{-x^3}{x^3 - 4x} = \lim_{x \rightarrow -\infty} \frac{-1}{1 - \frac{4}{x^2}} = \frac{-1}{1 - 0} = -1$$

Voor $x \rightarrow -\infty$ is de horizontale asymptoot de lijn $y = -1$.

b $f(x) = 1$ geeft $x > 0 \wedge \frac{x^3}{x^3 - 4x} = 1 \vee x < 0 \wedge \frac{-x^3}{x^3 - 4x} = 1$

$x > 0 \wedge x^3 = x^3 - 4x$ $x > 0 \wedge 4x = 0$ geen opl.	$x < 0 \wedge -x^3 = x^3 - 4x$ $x < 0 \wedge -2x^3 + 4x = 0$ $x < 0 \wedge -2x(x^2 - 2) = 0$ $x < 0 \wedge (x = 0 \vee x^2 = 2)$ $x < 0 \wedge (x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2})$ $x = -\sqrt{2}$
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$f(x) = -1$ geeft $x > 0 \wedge \frac{x^3}{x^3 - 4x} = -1 \vee x < 0 \wedge \frac{-x^3}{x^3 - 4x} = -1$

$x > 0 \wedge x^3 = -x^3 + 4x$ $x > 0 \wedge 2x^3 - 4x = 0$ $x > 0 \wedge 2x(x^2 - 2) = 0$ $x > 0 \wedge (x = 0 \vee x^2 = 2)$ $x > 0 \wedge (x = 0 \vee x = \sqrt{2} \vee x = -\sqrt{2})$ $x = \sqrt{2}$	$x < 0 \wedge -x^3 = -x^3 + 4x$ $x < 0 \wedge 4x = 0$ $x < 0 \wedge x = 0$ geen opl.
--	---

Dus $A(-\sqrt{2}, 1)$ en $B(\sqrt{2}, -1)$.

Stel $k: y = ax + b$ met $a = \frac{-1 - 1}{\sqrt{2} - (-\sqrt{2})} = \frac{-2}{2\sqrt{2}} = \frac{-1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$.

$y = -\frac{1}{2}x\sqrt{2} + b$
door $A(-\sqrt{2}, 1)$ $\left\{ \begin{array}{l} -\frac{1}{2} \cdot -\sqrt{2} \cdot \sqrt{2} + b = 1 \\ -\frac{1}{2} \cdot -2 + b = 1 \\ 1 + b = 1 \\ b = 0 \end{array} \right.$

Dus $k: y = -\frac{1}{2}x\sqrt{2}$.

c $f(x) = 1\frac{1}{3}$ geeft $x > 0 \wedge \frac{x^3}{x^3 - 4x} = \frac{4}{3} \vee x < 0 \wedge \frac{-x^3}{x^3 - 4x} = \frac{4}{3}$

$x > 0 \wedge 4x^3 - 16x = 3x^3$ $x > 0 \wedge x^3 - 16x = 0$ $x > 0 \wedge x(x^2 - 16) = 0$ $x > 0 \wedge (x = 0 \vee x^2 = 16)$ $x > 0 \wedge (x = 0 \vee x = 4 \vee x = -4)$ $x = 4$	$x < 0 \wedge 4x^3 - 16x = -3x^3$ $x < 0 \wedge 7x^3 - 16x = 0$ $x < 0 \wedge x(7x^2 - 16) = 0$ $x < 0 \wedge (x = 0 \vee x^2 = \frac{16}{7})$ $x < 0 \wedge (x = 0 \vee x = \sqrt{\frac{16}{7}} \vee x = -\sqrt{\frac{16}{7}})$ $x = -\sqrt{\frac{16}{7}} = -\frac{4}{\sqrt{7}}$
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$f(x) > 1\frac{1}{3}$ geeft $-2 < x < -\frac{4}{\sqrt{7}} \vee 2 < x < 4$

$$35 \quad a \quad \left. \begin{array}{l} bx^2 - 18 = 0 \\ x = -3 \end{array} \right\} \begin{array}{l} 9b - 18 = 0 \\ b = 2 \end{array} \quad \left. \begin{array}{l} bx^2 - 18 = 0 \\ x = 3 \end{array} \right\} \begin{array}{l} 9b - 18 = 0 \\ b = 2 \end{array}$$

Dus $b = 2$.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{ax^2 + 5}{2x^2 - 18} = \lim_{x \rightarrow \infty} \frac{a + \frac{5}{x^2}}{2 - \frac{18}{x^2}} = \frac{a + 0}{2 - 0} = \frac{1}{2}a$$

$$\lim_{x \rightarrow \infty} f(x) = 6 \text{ geeft } \frac{1}{2}a = 6, \text{ dus } a = 12.$$

Dus $a = 12$ en $b = 2$.

$$b \quad \left. \begin{array}{l} ax^4 + bx^3 - 2 = 0 \\ x = -2 \end{array} \right\} 16a - 8b - 2 = 0$$

$$\left. \begin{array}{l} ax^4 + bx^3 - 2 = 0 \\ x = 2 \end{array} \right\} 16a + 8b - 2 = 0$$

$$\begin{cases} 16a - 8b - 2 = 0 \\ 16a + 8b - 2 = 0 \end{cases}$$

$$\frac{-16b}{-16b} = 0$$

$$b = 0$$

$$16a - 8b - 2 = 0 \left\{ \begin{array}{l} 16a - 2 = 0 \\ a = \frac{1}{8} \end{array} \right.$$

$$\lim_{x \rightarrow \infty} \frac{5x^3 - 2x^2 + 1}{\frac{1}{8}x^4 - 2} = \lim_{x \rightarrow \infty} \frac{\frac{5}{x} - \frac{2}{x^2} + \frac{1}{x^4}}{\frac{1}{8} - \frac{2}{x^4}} = \frac{0 - 0 + 0}{\frac{1}{8} - 0} = 0, \text{ dus de lijn } y = 0 \text{ is}$$

horizontale asymptoot klopt.

Dus $a = \frac{1}{8}$ en $b = 0$.

Bladzijde 28

$$36 \quad f(x) = 0 \text{ geeft } x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = 3 \quad \vee \quad x = -3$$

vold. niet

Dus $A(-3, 0)$.

$$x^2 - 3x = 0 \wedge x^2 - 9 \neq 0$$

$$x(x - 3) = 0 \wedge x^2 \neq 9$$

$$(x = 0 \vee x = 3) \wedge x \neq 3 \wedge x \neq -3$$

$$x = 0$$

De verticale asymptoot is de lijn $x = 0$.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x^2 - 9}{x^2 - 3x} = \frac{1 - \frac{9}{x^2}}{1 - \frac{3}{x}} = \frac{1 - 0}{1 - 0} = 1$$

De horizontale asymptoot is de lijn $y = 1$.

Dus $B(0, 1)$.

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 3x} = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{x(x - 3)} = \lim_{x \rightarrow 3} \frac{x + 3}{x} = \frac{3 + 3}{3} = 2$$

Dus de perforatie is $(3, 2)$.

$$\text{Stel } k: y = ax + 1 \text{ met } a = \frac{1 - 0}{0 - -3} = \frac{1}{3}.$$

$$\text{Dus } k: y = \frac{1}{3}x + 1.$$

$$k: y = \frac{1}{3}x + 1 \left\{ \frac{1}{3} \cdot 3 + 1 = 2 \text{ klopt.} \right.$$

door $(3, 2)$?

Dus de lijn k gaat door de perforatie van de grafiek van f .

37 $2 \sin(x) + 1 = 0 \wedge \cos(x) \neq 0$

$$2 \sin(x) = -1 \wedge x \neq \frac{1}{2}\pi + k \cdot \pi$$

$$\sin(x) = -\frac{1}{2} \wedge x \neq \frac{1}{2}\pi + k \cdot \pi$$

$$(x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{1}{6}\pi + k \cdot 2\pi) \wedge x \neq \frac{1}{2}\pi + k \cdot \pi$$

De verticale asymptoten zijn de lijnen $x = \frac{1}{6}\pi$ en $x = \frac{5}{6}\pi$.

$$f(x) = 0 \text{ geeft } \cos(x) = 0, \text{ dus } x = \frac{1}{2}\pi + k \cdot \pi.$$

Dus $A(\frac{1}{2}\pi, 0)$.

$$f(x) = \frac{\cos(x)}{2 \sin(x) + 1} \text{ geeft}$$

$$f'(x) = \frac{(2 \sin(x) + 1) \cdot -\sin(x) - \cos(x) \cdot 2 \cos(x)}{(2 \sin(x) + 1)^2} = \frac{-2 \sin^2(x) - \sin(x) - 2 \cos^2(x)}{(2 \sin(x) + 1)^2} = \frac{-2 - \sin(x)}{(2 \sin(x) + 1)^2}$$

$$\text{Stel } k: y = ax + b \text{ met } a = f'(\frac{1}{2}\pi) = \frac{-2 - 1}{(2 + 1)^2} = -\frac{1}{3}.$$

$$\left. \begin{array}{l} y = -\frac{1}{3}x + b \\ \text{door } A(\frac{1}{2}\pi, 0) \end{array} \right\} -\frac{1}{3} \cdot \frac{1}{2}\pi + b = 0$$

$$b = \frac{1}{6}\pi$$

Dus $k: y = -\frac{1}{3}x + \frac{1}{6}\pi$.

$$x = \frac{1}{6}\pi \text{ geeft } y = -\frac{1}{3} \cdot \frac{1}{6}\pi + \frac{1}{6}\pi = -\frac{2}{9}\pi$$

$$x = \frac{5}{6}\pi \text{ geeft } y = -\frac{1}{3} \cdot \frac{5}{6}\pi + \frac{1}{6}\pi = -\frac{4}{9}\pi$$

Dus $B(\frac{1}{6}\pi, -\frac{2}{9}\pi)$ en $C(\frac{5}{6}\pi, -\frac{4}{9}\pi)$.

38 a $f(x) = \frac{\cos^2(x)}{\cos(x) + 1}$ geeft $f'(x) = \frac{(\cos(x) + 1) \cdot 2 \cos(x) \cdot -\sin(x) - \cos^2(x) \cdot -\sin(x)}{(\cos(x) + 1)^2}$

$$= \frac{-2 \sin(x) \cos^2(x) - 2 \sin(x) \cos(x) + \sin(x) \cos^2(x)}{(\cos(x) + 1)^2}$$

$$= \frac{-\sin(x) \cos^2(x) - 2 \sin(x) \cos(x)}{(\cos(x) + 1)^2}$$

$$f'(x) = 0 \text{ geeft } -\sin(x) \cos^2(x) - 2 \sin(x) \cos(x) = 0$$

$$-\sin(x) \cos(x) (\cos(x) + 2) = 0$$

$$\sin(x) = 0 \vee \cos(x) = 0 \vee \cos(x) = -2$$

$$x = \frac{1}{2}\pi + k \cdot \pi \vee x = k \cdot \pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = \frac{1}{2}\pi \vee x = \pi \text{ (vold. niet)} \vee x = \frac{3}{2}\pi \vee x = 2\pi$$

$$\text{max. is } f(0) = \frac{1}{2}$$

$$\text{min. is } f(\frac{1}{2}\pi) = 0$$

$$\text{min. is } f(\frac{3}{2}\pi) = 0$$

$$\text{max. is } f(2\pi) = \frac{1}{2}$$

b $\cos(x) + 1 = 0 \wedge \cos^2(x) \neq 0$ met $x \text{ in } [0, 2\pi]$ geeft $x = \pi$.

De verticale asymptoot is de lijn $x = \pi$.

$$f(x) = \frac{1}{2} \text{ geeft } \frac{\cos^2(x)}{\cos(x) + 1} = \frac{1}{2}$$

$$2 \cos^2(x) = \cos(x) + 1$$

$$2 \cos^2(x) - \cos(x) - 1 = 0$$

$$\text{Stel } \cos(x) = u.$$

$$2u^2 - u - 1 = 0$$

$$D = (-1)^2 - 4 \cdot 2 \cdot -1 = 9$$

$$u = \frac{1 \pm 3}{4} = 1 \vee u = \frac{1 - 3}{4} = -\frac{1}{2}$$

$$\cos(x) = 1 \vee \cos(x) = -\frac{1}{2}$$

$$x = k \cdot 2\pi \vee x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = \frac{4}{3}\pi + k \cdot 2\pi$$

$$x \text{ in } [0, 2\pi] \text{ geeft } x = 0 \vee x = \frac{2}{3}\pi \vee x = \frac{4}{3}\pi \vee x = 2\pi$$

Dus $A(\frac{2}{3}\pi, \frac{1}{2})$ en de afstand van A tot de lijn $x = \pi$ is $\frac{1}{3}\pi$.

39 a $x = \pi$ geeft $\sqrt{3} - 2 \sin(a\pi) = 0 \wedge \cos(a\pi) \neq 0$
 $2 \sin(a\pi) = \sqrt{3} \wedge a\pi \neq \frac{1}{2}\pi + k \cdot \pi$
 $\sin(a\pi) = \frac{1}{2}\sqrt{3} \wedge a \neq \frac{1}{2} + k \cdot 1$
 $(a\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee a\pi = \frac{2}{3}\pi + k \cdot 2\pi) \wedge a \neq \frac{1}{2} + k \cdot 1$
 $(a = \frac{1}{3} + k \cdot 2 \vee a = \frac{2}{3} + k \cdot 2) \wedge a \neq \frac{1}{2} + k \cdot 1$
 $a = \frac{1}{3} + k \cdot 2 \vee a = \frac{2}{3} + k \cdot 2$

$0 < a < 1$ geeft $a = \frac{1}{3} \vee a = \frac{2}{3}$

b Voor een perforatie moet gelden $\cos(ax) = 0 \wedge \sqrt{3} - 2 \sin(ax) = 0$
 $ax = \frac{1}{2}\pi + k \cdot \pi \wedge \sin(ax) = \frac{1}{2}\sqrt{3}$
 $ax = \frac{1}{2}\pi + k \cdot \pi \wedge (ax = \frac{1}{3}\pi + k \cdot 2\pi \vee ax = \frac{2}{3}\pi + k \cdot 2\pi)$
 Er bestaat geen waarde van a waarvoor de grafiek van f_a een perforatie heeft.

40 teller = 0 geeft $4 \sin(x) - 2 = 0$
 $4 \sin(x) = 2$
 $\sin(x) = \frac{1}{2}$
 $x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi$

noemer = 0 geeft $4 \cos^2(x) - 3 = 0$

$4 \cos^2(x) = 3$

$\cos^2(x) = \frac{3}{4}$

$\cos(x) = \frac{1}{2}\sqrt{3} \vee \cos(x) = -\frac{1}{2}\sqrt{3}$

$x = \frac{1}{6}\pi + k \cdot 2\pi \vee x = -\frac{1}{6}\pi + k \cdot 2\pi \vee x = \frac{5}{6}\pi + k \cdot 2\pi \vee x = -\frac{5}{6}\pi + k \cdot 2\pi$

x in $[0, 2\pi]$ geeft $x = \frac{1}{6}\pi \vee x = \frac{5}{6}\pi \vee x = \frac{7}{6}\pi \vee x = \frac{11}{6}\pi$

De verticale asymptoten zijn de lijnen $x = \frac{1}{6}\pi$ en $x = \frac{5}{6}\pi$.

$f(x) = \frac{4 \sin(x) - 2}{4 \cos^2(x) - 3} = \frac{4 \sin(x) - 2}{4(1 - \sin^2(x)) - 3} = \frac{4 \sin(x) - 2}{1 - 4 \sin^2(x)} = \frac{-2(1 - 2 \sin(x))}{(1 + 2 \sin(x))(1 - 2 \sin(x))}$

$\lim_{x \rightarrow \frac{1}{6}\pi} f(x) = \lim_{x \rightarrow \frac{1}{6}\pi} \frac{-2(1 - 2 \sin(x))}{(1 + 2 \sin(x))(1 - 2 \sin(x))} = \lim_{x \rightarrow \frac{1}{6}\pi} \frac{-2}{1 + 2 \sin(x)} = \frac{-2}{1 + 2 \cdot \frac{1}{2}} = -1$

$\lim_{x \rightarrow \frac{5}{6}\pi} f(x) = \lim_{x \rightarrow \frac{5}{6}\pi} \frac{-2(1 - 2 \sin(x))}{(1 + 2 \sin(x))(1 - 2 \sin(x))} = \lim_{x \rightarrow \frac{5}{6}\pi} \frac{-2}{1 + 2 \sin(x)} = \frac{-2}{1 + 2 \cdot \frac{1}{2}} = -1$

De perforaties zijn de punten $(\frac{1}{6}\pi, -1)$ en $(\frac{5}{6}\pi, -1)$.

Bladzijde 29

41 a $\lim_{x \rightarrow \infty} \frac{4}{x+2} = \lim_{x \rightarrow \infty} \frac{\frac{4}{x}}{1 + \frac{2}{x}} = \frac{0}{1+0} = 0$ en $\lim_{x \rightarrow -\infty} \frac{4}{x+2} = \lim_{x \rightarrow -\infty} \frac{\frac{4}{x}}{1 + \frac{2}{x}} = \frac{0}{1+0} = 0$

Dus de grafiek van f nadert de lijn $y = x + 3$ onbeperkt voor $x \rightarrow \infty$ en voor $x \rightarrow -\infty$.

b $\lim_{x \rightarrow \infty} h(x) = \lim_{x \rightarrow \infty} \frac{3}{x+1} = \lim_{x \rightarrow \infty} \frac{\frac{3}{x}}{1 + \frac{1}{x}} = \frac{0}{1+0} = 0$

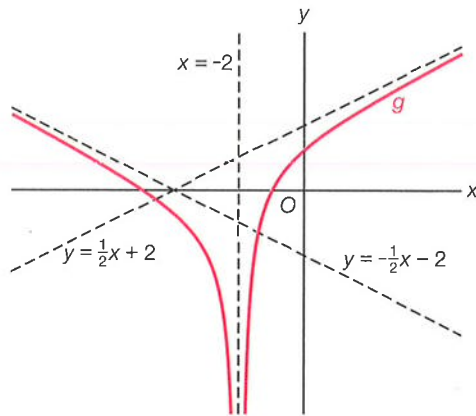
Dus de grafiek van s nadert de grafiek van g onbeperkt voor $x \rightarrow \infty$.

Bladzijde 30

42 a Voor $x < 2$ is $g(x) = \frac{x^2 + 6x + 5}{-2x - 4} = \frac{-\frac{1}{2}x(-2x - 4) - 2x + 6x + 5}{-2x - 4} = -\frac{1}{2}x + \frac{4x + 5}{-2x - 4}$
 $= -\frac{1}{2}x + \frac{-2(-2x - 4) - 8 + 5}{-2x - 4} = -\frac{1}{2}x - 2 + \frac{3}{2x + 4}$

b De scheve asymptoten zijn de lijnen $y = \frac{1}{2}x + 2$ en $y = -\frac{1}{2}x - 2$.

c



Bladzijde 31

43

a $x - 1 = 0 \wedge 2x^2 + 4x - 5 \neq 0$

$$x = 1 \wedge 2x^2 + 4x - 5 \neq 0$$

$$x = 1$$

De verticale asymptoot is de lijn $x = 1$.

$$f(x) = \frac{2x^2 + 4x - 5}{x - 1} = \frac{2x(x - 1) + 2x + 4x - 5}{x - 1} = 2x + \frac{6x - 5}{x - 1} = 2x + \frac{6(x - 1) + 6 - 5}{x - 1} = 2x + 6 + \frac{1}{x - 1}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x - 1} = 0, \text{ dus de scheve asymptoot is de lijn } y = 2x + 6.$$

b $2x - 1 = 0 \wedge -x^2 + 3x - 2 \neq 0$

$$2x = 1 \wedge -x^2 + 3x - 2 \neq 0$$

$$x = \frac{1}{2} \wedge -x^2 + 3x - 2 \neq 0$$

$$x = \frac{1}{2}$$

De verticale asymptoot is de lijn $x = \frac{1}{2}$.

$$g(x) = \frac{-x^2 + 3x - 2}{2x - 1} = \frac{-\frac{1}{2}x(2x - 1) - \frac{1}{2}x + 3x - 2}{2x - 1} = -\frac{1}{2}x + \frac{2\frac{1}{2}x - 2}{2x - 1} = -\frac{1}{2}x + \frac{1\frac{1}{4}(2x - 1) + 1\frac{1}{4} - 2}{2x - 1}$$

$$= -\frac{1}{2}x + 1\frac{1}{4} - \frac{\frac{3}{4}}{2x - 1}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{3}{4}}{2x - 1} = 0, \text{ dus de scheve asymptoot is de lijn } y = -\frac{1}{2}x + 1\frac{1}{4}.$$

c $x^2 - 9 = 0 \wedge x^3 + 3x^2 - 9x + 31 \neq 0$

$$x^2 = 9 \wedge x^3 + 3x^2 - 9x + 31 \neq 0$$

$$(x = 3 \vee x = -3) \wedge x^3 + 3x^2 - 9x + 31 \neq 0$$

$$x = 3 \vee x = -3$$

De verticale asymptoten zijn de lijnen $x = 3$ en $x = -3$.

$$h(x) = \frac{x^3 + 3x^2 - 9x + 31}{x^2 - 9} = \frac{x(x^2 - 9) + 9x + 3x^2 - 9x + 31}{x^2 - 9} = x + \frac{3x^2 + 31}{x^2 - 9} = x + \frac{3(x^2 - 9) + 27 + 31}{x^2 - 9}$$

$$= x + 3 + \frac{58}{x^2 - 9}$$

$$\lim_{x \rightarrow \infty} \frac{58}{x^2 - 9} = 0, \text{ dus de scheve asymptoot is de lijn } y = x + 3.$$

d $x^2 - 1 = 0 \wedge x^3 - 1 \neq 0$

$$x^2 = 1 \wedge x^3 \neq 1$$

$$(x = 1 \vee x = -1) \wedge x \neq 1$$

$$x = -1$$

De verticale asymptoot is de lijn $x = -1$.

$$j(x) = \frac{x^3 - 1}{x^2 - 1} = \frac{x(x^2 - 1) + x - 1}{x^2 - 1} = x + \frac{x - 1}{x^2 - 1}$$

$$\lim_{x \rightarrow \infty} \frac{x - 1}{x^2 - 1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 - \frac{1}{x^2}} = \frac{0 - 0}{1 - 0} = 0, \text{ dus de scheve asymptoot is de lijn } y = x.$$

44 a $x + 2 = 0 \wedge x^2 - x - 2 \neq 0$
 $x = -2$

De verticale asymptoot is de lijn $x = -2$.

$$f(x) = \frac{x^2 - x - 2}{x + 2} = \frac{x(x + 2) - 2x - x - 2}{x + 2} = x + \frac{-3x - 2}{x + 2} = x + \frac{-3(x + 2) + 6 - 2}{x + 2} = x - 3 + \frac{4}{x + 2}$$

$$\lim_{x \rightarrow \infty} \frac{4}{x + 2} = 0, \text{ dus de scheve asymptoot is de lijn } y = x - 3.$$

b $f(x) = x - 3 + \frac{4}{x + 2} = x - 3 + 4(x + 2)^{-1}$ geeft $f'(x) = 1 - 4(x + 2)^{-2} = 1 - \frac{4}{(x + 2)^2}$

$$f'(x) = 0 \text{ geeft } 1 - \frac{4}{(x + 2)^2} = 0$$

$$\frac{4}{(x + 2)^2} = 1$$

$$(x + 2)^2 = 4$$

$$x + 2 = 2 \vee x + 2 = -2$$

$$x = 0 \vee x = -4$$

$$\text{max. is } f(-4) = -9 \text{ en min. is } f(0) = -1$$

c $f(x) = 0$ geeft $x^2 - x - 2 = 0$

$$(x + 1)(x - 2) = 0$$

$$x = -1 \vee x = 2$$

$$f(x) \leq 0 \text{ geeft } x < -2 \vee -1 \leq x \leq 2$$

45 a $|x| = 0 \wedge x^2 + 3x + 2 \neq 0$
 $x = 0$

De verticale asymptoot is de lijn $x = 0$.

$$\text{Voor } x > 0 \text{ is } f(x) = \frac{x^2 + 3x + 2}{x} = x + 3 + \frac{2}{x}.$$

$$\lim_{x \rightarrow \infty} \frac{2}{x} = 0, \text{ dus de lijn } y = x + 3 \text{ is scheve asymptoot.}$$

$$\text{Voor } x < 0 \text{ is } f(x) = \frac{x^2 + 3x + 2}{-x} = -x - 3 - \frac{2}{x}.$$

$$\lim_{x \rightarrow -\infty} \frac{2}{x} = 0, \text{ dus de lijn } y = -x - 3 \text{ is scheve asymptoot.}$$

b Voor $x > 0$ is $f(x) = x + 3 + \frac{2}{x} = x + 3 + 2x^{-1}$ en $f'(x) = 1 - 2x^{-2} = 1 - \frac{2}{x^2}$.

$$f'(x) = 0 \text{ geeft } 1 - \frac{2}{x^2} = 0$$

$$\frac{2}{x^2} = 1$$

$$x^2 = 2$$

$$x = \sqrt{2} \vee x = -\sqrt{2}$$

vold. niet

$$\text{Voor } x < 0 \text{ is } f(x) = -x - 3 - \frac{2}{x} = -x - 3 - 2x^{-1} \text{ en } f'(x) = -1 + 2x^{-2} = -1 + \frac{2}{x^2}.$$

$$f'(x) = 0 \text{ geeft } -1 + \frac{2}{x^2} = 0$$

$$\frac{2}{x^2} = 1$$

$$x^2 = 2$$

$$x = \sqrt{2} \vee x = -\sqrt{2}$$

vold. niet

$$\text{min. is } f(-\sqrt{2}) = \frac{4 - 3\sqrt{2}}{\sqrt{2}} = 2\sqrt{2} - 3 \text{ en min. is } f(\sqrt{2}) = \frac{4 + 3\sqrt{2}}{\sqrt{2}} = 2\sqrt{2} + 3$$

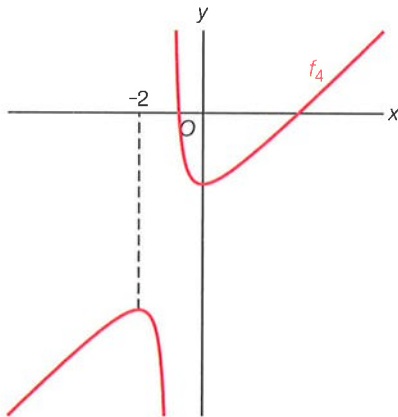
c Voor $x < 0$ geldt $f(x) = 6$ geeft $\frac{x^2 + 3x + 2}{-x} = 6$
 $x^2 + 3x + 2 = -6x$
 $x^2 + 9x + 2 = 0$
 $D = 9^2 - 4 \cdot 1 \cdot 2 = 73$
 $x = \frac{-9 + \sqrt{73}}{2} = -4\frac{1}{2} + \frac{1}{2}\sqrt{73} \vee x = -4\frac{1}{2} - \frac{1}{2}\sqrt{73}$

Voor $x > 0$ geldt $f(x) = 6$ geeft $\frac{x^2 + 3x + 2}{x} = 6$
 $x^2 + 3x + 2 = 6x$
 $x^2 - 3x + 2 = 0$
 $(x - 1)(x - 2) = 0$
 $x = 1 \vee x = 2$
 $f(x) \leq 6$ geeft $-4\frac{1}{2} - \frac{1}{2}\sqrt{73} \leq x \leq -4\frac{1}{2} + \frac{1}{2}\sqrt{73} \vee 1 \leq x \leq 2$

46 a $f_2(x) = \frac{4x^2 - 9x - 9}{4x + 2} = \frac{x(4x + 2) - 2x - 9x - 9}{4x + 2} = x + \frac{-11x - 9}{4x + 2} = x + \frac{-\frac{11}{4}(4x + 2) + 5\frac{1}{2} - 9}{4x + 2}$
 $= x - 2\frac{3}{4} - \frac{3\frac{1}{2}}{4x + 2}$

$\lim_{x \rightarrow \infty} \frac{3\frac{1}{2}}{4x + 2} = 0$, dus de scheve asymptoot is de lijn $y = x - 2\frac{3}{4}$.

b $f_4(x) = \frac{4x^2 - 9x - 9}{4x + 4}$ geeft
 $f_4'(x) = \frac{(4x + 4) \cdot (8x - 9) - (4x^2 - 9x - 9) \cdot 4}{(4x + 4)^2} = \frac{32x^2 - 36x + 32x - 36 - 16x^2 + 36x + 36}{(4x + 4)^2} = \frac{16x^2 + 32x}{(4x + 4)^2}$
 $f_4'(x) = 0$ geeft $16x^2 + 32x = 0$
 $16x(x + 2) = 0$
 $x = 0 \vee x = -2$



max. is $f_4(-2) = -6\frac{1}{4}$ en min. is $f_4(0) = -2\frac{1}{4}$

c $f_a(x) = \frac{4x^2 - 9x - 9}{4x + a} = \frac{(x - 3)(4x + 3)}{4x + a} = \frac{\frac{1}{4}(4x - 12)(4x + 3)}{4x + a}$

Er is een perforatie als $a = -12$ en als $a = 3$.

$\lim_{x \rightarrow 3} f_{-12}(x) = \lim_{x \rightarrow 3} \frac{\frac{1}{4}(4x - 12)(4x + 3)}{4x - 12} = \lim_{x \rightarrow 3} \frac{1}{4}(4x + 3) = 3\frac{3}{4}$

Dus voor $a = -12$ is de perforatie $(3, 3\frac{3}{4})$.

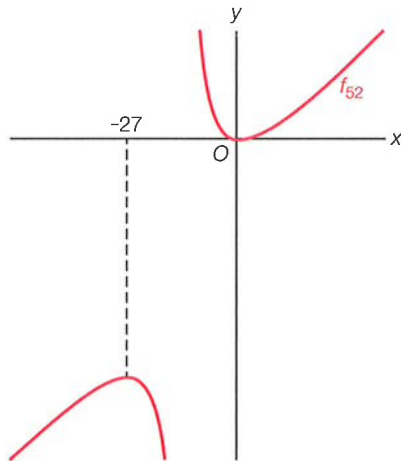
$\lim_{x \rightarrow -\frac{3}{4}} f_3(x) = \lim_{x \rightarrow -\frac{3}{4}} \frac{\frac{1}{4}(4x - 12)(4x + 3)}{4x + 3} = \lim_{x \rightarrow -\frac{3}{4}} \frac{1}{4}(4x - 12) = \lim_{x \rightarrow -\frac{3}{4}} (x - 3) = -3\frac{3}{4}$

Dus voor $a = 3$ is de perforatie $(-\frac{3}{4}, -3\frac{3}{4})$.

47 $f_a(x) = \frac{4x^2 - 9x - 9}{4x + a}$ geeft $f'_a(x) = \frac{(4x + a) \cdot (8x - 9) - (4x^2 - 9x - 9) \cdot 4}{(4x + a)^2}$
 $f'_a(1) = 0$ geeft $(4 + a) \cdot (8 - 9) - (4 - 9 - 9) \cdot 4 = 0$
 $-4 - a + 56 = 0$
 $a = 52$

$$f_{52}'(x) = \frac{(4x + 52) \cdot (8x - 9) - (4x^2 - 9x - 9) \cdot 4}{(4x + 52)^2}$$

$f_{52}'(x) = 0$ geeft $(4x + 52) \cdot (8x - 9) - (4x^2 - 9x - 9) \cdot 4 = 0$
 $32x^2 - 36x + 416x - 468 - 16x^2 + 36x + 36 = 0$
 $16x^2 + 416x - 432 = 0$
 $x^2 + 26x - 27 = 0$
 $(x - 1)(x + 27) = 0$
 $x = 1 \vee x = -27$



max. is $f_{52}(-27) = -56\frac{1}{4}$

Bladzijde 32

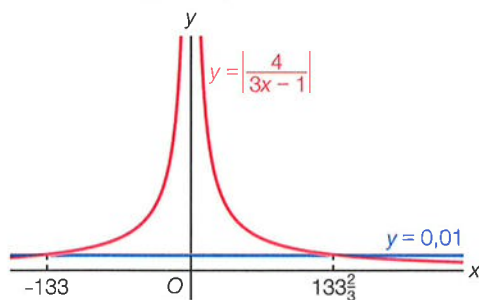
48 a $f_0(x) = \frac{3x^2 + 11x}{3x - 1} = \frac{x(3x - 1) + x + 11x}{3x - 1} = x + \frac{12x}{3x - 1} = x + \frac{4(3x - 1) + 4}{3x - 1} = x + 4 + \frac{4}{3x - 1}$

$\lim_{x \rightarrow \infty} \frac{4}{3x - 1} = 0$, dus de scheve asymptoot is de lijn $k: y = x + 4$.

$|f_0(x) - (x + 4)| < 0,01$ geeft $\left| x + 4 + \frac{4}{3x - 1} - x - 4 \right| < 0,01$ oftewel $\left| \frac{4}{3x - 1} \right| < 0,01$.

Voer in $y_1 = \left| \frac{4}{3x - 1} \right|$ en $y_2 = 0,01$.

De optie snijpunt geeft $x = -133$ en $x = 133\frac{2}{3}$.



$|f_0(x) - (x + 4)| < 0,01$ geeft $x < -133 \vee x > 133\frac{2}{3}$

$$b \quad f_a(x) = \frac{(3x-1)(x-a)}{3x-1} = \frac{3x^2 - 3ax - x + a}{3x-1} = \frac{3x^2 + (-3a-1)x + a}{3x-1}$$

$$\text{Dus } -3a - 1 = 11$$

$$-3a = 12$$

$$a = -4$$

$$\lim_{x \rightarrow \frac{1}{3}} f_{-4}(x) = \lim_{x \rightarrow \frac{1}{3}} \frac{3x^2 + 11x - 4}{3x-1} = \lim_{x \rightarrow \frac{1}{3}} \frac{(3x-1)(x+4)}{3x-1} = \lim_{x \rightarrow \frac{1}{3}} (x+4) = 4\frac{1}{3}$$

Dus de perforatie is het punt $(\frac{1}{3}, 4\frac{1}{3})$.

$$49 \quad a \quad f(x) = \frac{x^3 - 6x^2 + 8x}{x^2 + x - 6} = \frac{x(x^2 - 6x + 8)}{(x-2)(x+3)} = \frac{x(x-2)(x-4)}{(x-2)(x+3)}$$

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{x(x-2)(x-4)}{(x-2)(x+3)} = \lim_{x \rightarrow 2} \frac{x(x-4)}{x+3} = \frac{2(2-4)}{2+3} = -\frac{4}{5}$$

Dus de perforatie is $A(2, -\frac{4}{5})$.

$$(x-2)(x+3) = 0 \wedge x(x-2)(x-4) \neq 0$$

$$(x=2 \vee x=-3) \wedge x \neq 0 \wedge x \neq 2 \wedge x \neq 4$$

$$x = -3$$

Dus de verticale asymptoot is de lijn $x = -3$.

$$f(x) = \frac{x(x-4)}{x+3} = \frac{x^2 - 4x}{x+3} = \frac{x(x+3) - 3x - 4x}{x+3} = x - \frac{7x}{x+3} = x - \frac{7(x+3) - 21}{x+3} = x - 7 + \frac{21}{x+3}$$

$$\lim_{x \rightarrow \infty} \frac{21}{x+3} = 0, \text{ dus de scheve asymptoot is de lijn } y = x - 7.$$

$$\left. \begin{array}{l} y = x - 7 \\ x = -3 \end{array} \right\} y = -3 - 7 = -10$$

Dus $B(-3, -10)$.

$$\text{Stel } k: y = ax + b \text{ met } a = \frac{-\frac{4}{5} - (-10)}{2 - (-3)} = \frac{46}{25}.$$

$$\left. \begin{array}{l} y = \frac{46}{25}x + b \\ \text{door } B(-3, -10) \end{array} \right\} \begin{array}{l} \frac{46}{25} \cdot (-3) + b = -10 \\ -\frac{138}{25} + b = -10 \\ b = -4\frac{12}{25} \end{array}$$

Dus $k: y = 1\frac{21}{25}x - 4\frac{12}{25}$ en het snijpunt met de y -as is $C(0, -4\frac{12}{25})$.

$$b \quad g(x) = \frac{x(x-4)}{x+3} = \frac{x^2 - 4x}{x+3} \text{ geeft}$$

$$g'(x) = \frac{(x+3)(2x-4) - (x^2 - 4x) \cdot 1}{(x+3)^2} = \frac{2x^2 - 4x + 6x - 12 - x^2 + 4x}{(x+3)^2} = \frac{x^2 + 6x - 12}{(x+3)^2}$$

$$\text{De helling in } A \text{ is } g'(2) = \frac{4 + 12 - 12}{5^2} = \frac{4}{25}, \text{ dus } rc_l = -\frac{25}{4} = -6\frac{1}{4}.$$

$$\left. \begin{array}{l} y = -6\frac{1}{4}x + b \\ \text{door } A(2, -\frac{4}{5}) \end{array} \right\} \begin{array}{l} -6\frac{1}{4} \cdot 2 + b = -\frac{4}{5} \\ -12\frac{1}{2} + b = -\frac{4}{5} \\ b = 11\frac{7}{10} \end{array}$$

Dus $l: y = -6\frac{1}{4}x + 11\frac{7}{10}$ en $D(0, 11\frac{7}{10})$.

13.4 Limieten bij exponentiële en logaritmische functies

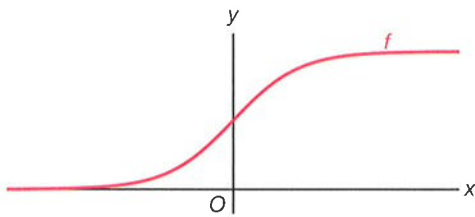
Bladzijde 34

50

a

x	-6	-3	0	3	6
$f(x)$	0,01	0,28	3	5,72	5,99

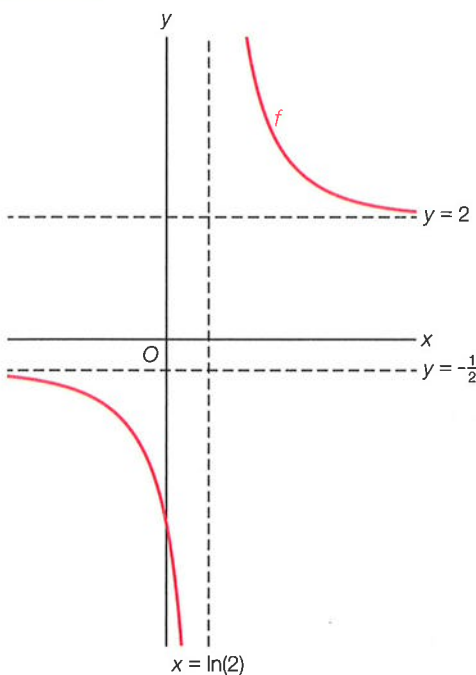
b

c De lijnen $y = 0$ en $y = 6$.

Bladzijde 35

51

a



b Voor $x \rightarrow \infty$ krijg je $y = 3x - 4 + 2$, dus $y = 3x - 2$.
 Voor $x \rightarrow -\infty$ krijg je $y = 3x - 4 - \frac{1}{2}$, dus $y = 3x - 4\frac{1}{2}$.

52

a

$$\lim_{x \rightarrow \infty} (10 - 4 \cdot (\frac{1}{2})^x) = 10 - 4 \cdot 0 = 10$$

b

$$\lim_{x \rightarrow \infty} (2 + 4e^{-x}) = 2 + 4 \cdot 0 = 2$$

c

$$\lim_{x \rightarrow -\infty} \frac{e^x - 3}{e^x + 2} = \frac{0 - 3}{0 + 2} = -1\frac{1}{2}$$

d

$$\lim_{x \rightarrow -\infty} \frac{2 \cdot 3^x + 1}{3 \cdot 2^x + 1} = \frac{2 \cdot 0 + 1}{3 \cdot 0 + 1} = 1$$

e

$$\lim_{x \rightarrow \infty} \frac{4e^x - 3}{2 - e^x} = \lim_{x \rightarrow \infty} \frac{4 - \frac{3}{e^x}}{\frac{2}{e^x} - 1} = \frac{4 - 0}{0 - 1} = -4$$

f

$$\lim_{x \rightarrow \infty} \frac{2^x + 16}{4^x - 1} = \lim_{x \rightarrow \infty} \frac{\frac{2^x}{4^x} + \frac{16}{4^x}}{1 - \frac{1}{4^x}} = \lim_{x \rightarrow \infty} \frac{(\frac{1}{2})^x + \frac{16}{4^x}}{1 - \frac{1}{4^x}} = \frac{0 + 0}{1 - 0} = 0$$

Bladzijde 36

53 a $e^x - e = 0 \wedge 5e^x \neq 0$

$$e^x = e^1$$

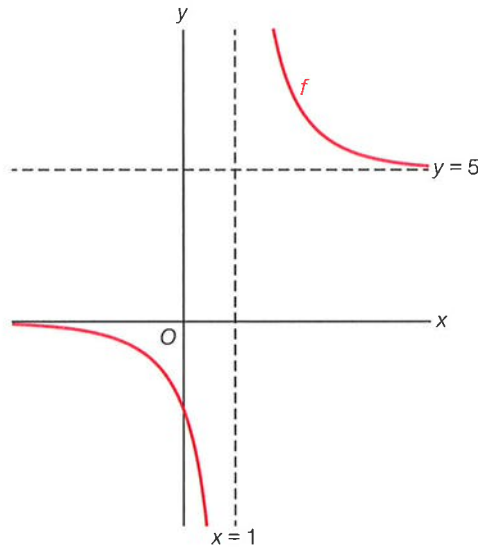
$$x = 1$$

De verticale asymptoot is de lijn $x = 1$.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{5e^x}{e^x - e} = \lim_{x \rightarrow \infty} \frac{5}{1 - \frac{e}{e^x}} = \frac{5}{1 - 0} = 5$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{5e^x}{e^x - e} = \frac{0}{0 - e} = 0$$

De horizontale asymptoten zijn de lijnen $y = 5$ en $y = 0$.



b Voor $0 \leq p \leq 5$ heeft de vergelijking $f(x) = p$ geen oplossingen.

c $f(x) = \frac{5e^x}{e^x - e}$ geeft $f'(x) = \frac{(e^x - e) \cdot 5e^x - 5e^x \cdot e^x}{(e^x - e)^2} = \frac{5e^{2x} - 5e^{x+1} - 5e^{2x}}{(e^x - e)^2} = \frac{-5e^{x+1}}{(e^x - e)^2}$

$$rc_k = f'(0) = \frac{-5e^1}{(e^0 - e)^2} = \frac{-5e}{(1 - e)^2}$$

54 a Geen verticale asymptoot als $e^x + q = 0 \wedge -e^x - 1 \neq 0$ geen oplossing heeft.

Dat is het geval als $q \geq 0$.

b $\lim_{x \rightarrow -\infty} f_{p,q}(x) = -3$

$$\lim_{x \rightarrow -\infty} \frac{pe^x - 1}{e^x + q} = -3$$

$$\frac{p \cdot 0 - 1}{0 + q} = -3$$

$$\frac{-1}{q} = -3$$

$$q = \frac{1}{3}$$

$$\lim_{x \rightarrow \infty} f_{p,q}(x) = 4$$

$$\lim_{x \rightarrow \infty} \frac{pe^x - 1}{e^x + q} = 4$$

$$\lim_{x \rightarrow \infty} \frac{p - \frac{1}{e^x}}{1 + \frac{q}{e^x}} = 4$$

$$\frac{p - 0}{1 + 0} = 4$$

$$p = 4$$

Dus voor $p = 4$ en $q = \frac{1}{3}$.

c $e^2 + q = 0$

$$q = -e^2$$

$$\lim_{x \rightarrow \infty} f_{p,q}(x) = 2$$

$$\lim_{x \rightarrow \infty} \frac{pe^x - 1}{e^x + q} = 2$$

$$\lim_{x \rightarrow \infty} \frac{p - \frac{1}{e^x}}{1 + \frac{q}{e^x}} = 2$$

$$\frac{p - 0}{1 + 0} = 2$$

$$p = 2$$

Dus voor $p = 2$ en $q = -e^2$.

$$\text{Dit geeft } f_{2,-e^2}(x) = \frac{2e^x - 1}{e^x - e^2}.$$

$$\lim_{x \rightarrow -\infty} f_{2,-e^2}(x) = \lim_{x \rightarrow -\infty} \frac{2e^x - 1}{e^x - e^2} = \frac{2 \cdot 0 - 1}{0 - e^2} = \frac{1}{e^2}$$

Dus in dit geval is voor $x \rightarrow -\infty$ de horizontale asymptoot de lijn $y = \frac{1}{e^2}$.

d Er moet voor $x = -\ln(2)$ gelden dat $pe^x - 1 = 0 \wedge e^x + q = 0$.

$$\text{Dit geeft } pe^{-\ln(2)} - 1 = 0 \wedge e^{-\ln(2)} + q = 0$$

$$pe^{\ln(\frac{1}{2})} - 1 = 0 \wedge e^{\ln(\frac{1}{2})} + q = 0$$

$$p \cdot \frac{1}{2} - 1 = 0 \wedge \frac{1}{2} + q = 0$$

$$p = 2 \wedge q = -\frac{1}{2}$$

55 a $f(x) = \frac{2xe^x + 3}{e^{x+2}}$ geeft

$$f'(x) = \frac{e^{x+2} \cdot (2 \cdot e^x + 2x \cdot e^x) - (2xe^x + 3) \cdot e^{x+2}}{(e^{x+2})^2} = \frac{2e^x + 2xe^x - 2xe^x - 3}{e^{x+2}} = \frac{2e^x - 3}{e^{x+2}}$$

$$f'(x) = 0 \text{ geeft } 2e^x - 3 = 0$$

$$2e^x = 3$$

$$e^x = 1\frac{1}{2}$$

$$x = \ln(1\frac{1}{2})$$

$$f(\ln(1\frac{1}{2})) = \frac{2 \ln(1\frac{1}{2}) e^{\ln(1\frac{1}{2})} + 3}{e^{\ln(1\frac{1}{2}) + 2}} = \frac{2 \ln(1\frac{1}{2}) \cdot 1\frac{1}{2} + 3}{e^{\ln(1\frac{1}{2})} \cdot e^2} = \frac{3 \ln(1\frac{1}{2}) + 3}{1\frac{1}{2} e^2} = \frac{2 \ln(1\frac{1}{2}) + 2}{e^2}$$

$$\text{Dus } A\left(\ln(1\frac{1}{2}), \frac{2 \ln(1\frac{1}{2}) + 2}{e^2}\right).$$

b $f(x) = \frac{2xe^x + 3}{e^{x+2}} = \frac{2xe^x + 3}{e^x \cdot e^2} = \frac{2xe^x}{e^2 \cdot e^x} + \frac{3}{e^2 \cdot e^x} = \frac{2}{e^2}x + \frac{3}{e^2} \cdot \frac{1}{e^x}$

$$\lim_{x \rightarrow \infty} \frac{3}{e^2} \cdot \frac{1}{e^x} = \frac{3}{e^2} \cdot 0 = 0, \text{ dus de lijn } y = \frac{2}{e^2}x \text{ is scheve asymptoot.}$$

c $x = \ln(1\frac{1}{2})$ invullen bij $y = \frac{2}{e^2}x$ geeft $y = \frac{2}{e^2} \cdot \ln(1\frac{1}{2}) = \frac{2 \ln(1\frac{1}{2})}{e^2}$, dus

$$p = \frac{2 \ln(1\frac{1}{2}) + 2}{e^2} - \frac{2 \ln(1\frac{1}{2})}{e^2} = \frac{2}{e^2}.$$

$$\text{Los op } \left| f(x) - \frac{2}{e^2}x \right| = 0,01 \cdot \frac{2}{e^2} \text{ oftewel } \left| \frac{2xe^x + 3}{e^{x+2}} - \frac{2}{e^2}x \right| = \frac{0,02}{e^2}.$$

$$\text{Voer in } y_1 = \left| \frac{2xe^x + 3}{e^{x+2}} - \frac{2}{e^2}x \right| \text{ en } y_2 = \frac{0,02}{e^2}.$$

De optie snijpunt geeft $x \approx 5,01$.

56 a $\lim_{x \uparrow 0} f(x) = \lim_{\frac{1}{x} \rightarrow -\infty} \frac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1} = \frac{0}{0+1} = 0$

$\lim_{x \downarrow 0} f(x) = \lim_{\frac{1}{x} \rightarrow \infty} \frac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1} = \lim_{\frac{1}{x} \rightarrow \infty} \frac{2}{1 + \frac{1}{e^{\frac{1}{x}}}} = \frac{2}{1+0} = 2$

b $\lim_{x \rightarrow \infty} f(x) = \lim_{\frac{1}{x} \rightarrow 0} \frac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1} = \frac{2e^0}{e^0 + 1} = \frac{2}{1+1} = 1$

$\lim_{x \rightarrow -\infty} f(x) = \lim_{\frac{1}{x} \rightarrow 0} \frac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1} = \frac{2e^0}{e^0 + 1} = \frac{2}{1+1} = 1$

Er is dus één horizontale asymptoot, en de formule ervan is $y = 1$.

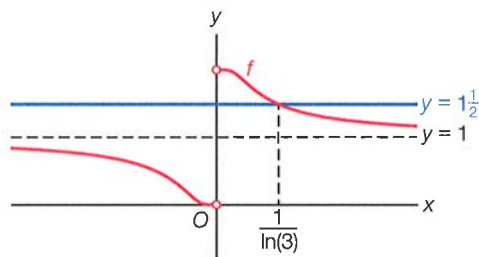
c $f(x) = 1\frac{1}{2}$ geeft $\frac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1} = \frac{3}{2}$

$4e^{\frac{1}{x}} = 3e^{\frac{1}{x}} + 3$

$e^{\frac{1}{x}} = 3$

$\frac{1}{x} = \ln(3)$

$x = \frac{1}{\ln(3)}$



$f(x) \leq 1\frac{1}{2}$ geeft $x < 0 \vee x \geq \frac{1}{\ln(3)}$

d $f(x) = \frac{2e^{\frac{1}{x}}}{e^{\frac{1}{x}} + 1}$ geeft

$f'(x) = \frac{(e^{\frac{1}{x}} + 1) \cdot 2e^{\frac{1}{x}} \cdot -\frac{1}{x^2} - 2e^{\frac{1}{x}} \cdot e^{\frac{1}{x}} \cdot -\frac{1}{x^2}}{(e^{\frac{1}{x}} + 1)^2} = \frac{2e^{\frac{2}{x}} + 2e^{\frac{1}{x}} - 2e^{\frac{2}{x}}}{(e^{\frac{1}{x}} + 1)^2} \cdot -\frac{1}{x^2} = \frac{-2e^{\frac{1}{x}}}{x^2(e^{\frac{1}{x}} + 1)^2}$

$rc_k = f'(1) = \frac{-2e^1}{1 \cdot (e^1 + 1)^2} = \frac{-2e}{(e+1)^2}$

Bladzijde 37

57 a $\ln(x) - 1 = 0 \wedge \ln(x) \neq 0$

$\ln(x) = 1$

$x = e$

De verticale asymptoot is de lijn $x = e$.

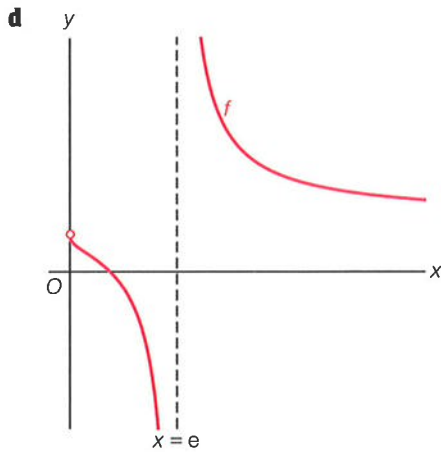
b $f(x) = 0$ geeft $\ln(x) = 0$

$x = 1$

Dus het nulpunt van f is 1.

c

x	0,00001	0,0001	10	100
$f(x)$	0,92	0,90	1,77	1,28



e $\lim_{x \downarrow 0} f(x) = 1$

f ja

Bladzijde 38

58 a $\lim_{x \rightarrow \infty} \frac{4 \ln(x)}{1 + \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{4 \ln(x)}{1 + \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{4}{\frac{1}{\ln(x)} + 1} = \frac{4}{0 + 1} = 4$

b $\lim_{x \rightarrow 0} \frac{\ln(x^2)}{1 + \ln(x^2)} = \lim_{x \rightarrow 0} \frac{2 \ln|x|}{1 + 2 \ln|x|} = \lim_{\ln|x| \rightarrow -\infty} \frac{2 \ln|x|}{1 + 2 \ln|x|} = \lim_{\ln|x| \rightarrow -\infty} \frac{2}{\frac{1}{\ln|x|} + 2} = \frac{2}{0 + 2} = 1$

c $\lim_{x \rightarrow -\infty} \frac{\ln|x|}{4 + \ln(x^2)} = \lim_{x \rightarrow -\infty} \frac{\ln|x|}{4 + 2 \ln|x|} = \lim_{\ln|x| \rightarrow -\infty} \frac{\ln|x|}{4 + 2 \ln|x|} = \lim_{\ln|x| \rightarrow -\infty} \frac{1}{\frac{4}{\ln|x|} + 2} = \frac{1}{0 + 2} = \frac{1}{2}$

d $\lim_{x \rightarrow \infty} \frac{\ln^2(x)}{2 - \ln^2(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{\ln^2(x)}{2 - \ln^2(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{1}{\frac{2}{\ln^2(x)} - 1} = \frac{1}{0 - 1} = -1$

59 a $\lim_{x \rightarrow \infty} \frac{3 + \ln(x)}{\ln(\sqrt{x})} = \lim_{x \rightarrow \infty} \frac{3 + \ln(x)}{\ln(x^{\frac{1}{2}})} = \lim_{x \rightarrow \infty} \frac{3 + \ln(x)}{\frac{1}{2} \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{3 + \ln(x)}{\frac{1}{2} \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{\frac{3}{\ln(x)} + 1}{\frac{1}{2}} = \frac{0 + 1}{\frac{1}{2}} = 2$

b $\lim_{x \rightarrow \infty} \frac{2^{\log(x)}}{1 + 8^{\log(x)}} = \lim_{x \rightarrow \infty} \frac{2^{\log(x)}}{1 + \frac{2^{\log(x)}}{3}} = \lim_{x \rightarrow \infty} \frac{2^{\log(x)}}{1 + \frac{2^{\log(x)}}{3}} = \lim_{2^{\log(x)} \rightarrow \infty} \frac{2^{\log(x)}}{1 + \frac{2^{\log(x)}}{3}} = \lim_{2^{\log(x)} \rightarrow \infty} \frac{1}{\frac{1}{2^{\log(x)}} + \frac{1}{3}} = \frac{1}{0 + \frac{1}{3}} = 3$

c $\lim_{x \rightarrow -\infty} \frac{\ln(x^2)}{1 - 2 \ln|x|} = \lim_{x \rightarrow -\infty} \frac{2 \ln|x|}{1 - 2 \ln|x|} = \lim_{\ln|x| \rightarrow -\infty} \frac{2 \ln|x|}{1 - 2 \ln|x|} = \lim_{\ln|x| \rightarrow -\infty} \frac{2}{\frac{1}{\ln|x|} - 2} = \frac{2}{0 - 2} = -1$

d $\lim_{x \rightarrow \infty} \frac{2 + \ln(x)}{2 + \log(x)} = \lim_{x \rightarrow \infty} \frac{2 + \ln(x)}{2 + \frac{\ln(x)}{\ln(10)}} = \lim_{\ln(x) \rightarrow \infty} \frac{2 + \ln(x)}{2 + \frac{\ln(x)}{\ln(10)}} = \lim_{\ln(x) \rightarrow \infty} \frac{\frac{2}{\ln(x)} + 1}{\frac{2}{\ln(x)} + \frac{1}{\ln(10)}} = \frac{0 + 1}{0 + \frac{1}{\ln(10)}} = \ln(10)$

Bladzijde 39

60 a $\lim_{x \downarrow 0} f(x) = \lim_{x \downarrow 0} \frac{\ln(x) - 1}{2 \ln(x) - 3} = \lim_{\ln(x) \rightarrow -\infty} \frac{\ln(x) - 1}{2 \ln(x) - 3} = \lim_{\ln(x) \rightarrow -\infty} \frac{1 - \frac{1}{\ln(x)}}{2 - \frac{3}{\ln(x)}} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$

b $2 \ln(x) - 3 = 0 \wedge \ln(x) - 1 \neq 0$
 $2 \ln(x) = 3 \wedge \ln(x) \neq 1$
 $\ln(x) = 1\frac{1}{2} \wedge \ln(x) \neq 1$
 $x = e^{1\frac{1}{2}} = e\sqrt{e}$

De verticale asymptoot is de lijn $x = e\sqrt{e}$.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{\ln(x) - 1}{2 \ln(x) - 3} = \lim_{\ln(x) \rightarrow \infty} \frac{1 - \frac{1}{\ln(x)}}{2 - \frac{3}{\ln(x)}} = \frac{1 - 0}{2 - 0} = \frac{1}{2}$$

De horizontale asymptoot is de lijn $y = \frac{1}{2}$.

Dus $B(e\sqrt{e}, \frac{1}{2})$.

$$f(x) = 0 \text{ geeft } \ln(x) - 1 = 0$$

$$\ln(x) = 1$$

$$x = e$$

Dus $A(e, 0)$.

$$f(x) = \frac{\ln(x) - 1}{2 \ln(x) - 3} \text{ geeft}$$

$$f'(x) = \frac{(2 \ln(x) - 3) \cdot \frac{1}{x} - (\ln(x) - 1) \cdot \frac{2}{x}}{(2 \ln(x) - 3)^2} = \frac{2 \ln(x) - 3 - 2 \ln(x) + 2}{x(2 \ln(x) - 3)^2} = \frac{-1}{x(2 \ln(x) - 3)^2}$$

$$\text{Stel } k: y = ax + b \text{ met } a = f'(e) = \frac{-1}{e(2 \ln(e) - 3)^2} = \frac{-1}{e(2 - 3)^2} = -\frac{1}{e}.$$

$$y = -\frac{1}{e}x + b \left\{ \begin{array}{l} -\frac{1}{e} \cdot e + b = 0 \\ \text{door } A(e, 0) \quad -1 + b = 0 \\ b = 1 \end{array} \right.$$

$$\text{Dus } k: y = -\frac{1}{e}x + 1.$$

$$k \text{ snijden met de horizontale asymptoot geeft } -\frac{1}{e}x + 1 = \frac{1}{2}$$

$$-\frac{1}{e}x = -\frac{1}{2}$$

$$x = \frac{1}{2}e$$

Dus $C(\frac{1}{2}e, \frac{1}{2})$.

$$k \text{ snijden met de verticale asymptoot geeft } y = -\frac{1}{e} \cdot e\sqrt{e} + 1 = 1 - \sqrt{e}, \text{ dus } D(e\sqrt{e}, 1 - \sqrt{e}).$$

$$B(e\sqrt{e}, \frac{1}{2}) \text{ en } C(\frac{1}{2}e, \frac{1}{2}) \text{ geeft } BC = e\sqrt{e} - \frac{1}{2}e$$

$$B(e\sqrt{e}, \frac{1}{2}) \text{ en } D(e\sqrt{e}, 1 - \sqrt{e}) \text{ geeft } BD = \frac{1}{2} - (1 - \sqrt{e}) = \sqrt{e} - \frac{1}{2}$$

$$O(\triangle BCD) = \frac{1}{2} \cdot (e\sqrt{e} - \frac{1}{2}e) \cdot (\sqrt{e} - \frac{1}{2}) = \frac{1}{2}(e^2 - \frac{1}{2}e\sqrt{e} - \frac{1}{2}e\sqrt{e} + \frac{1}{4}e) = \frac{1}{2}e^2 - \frac{1}{2}e\sqrt{e} + \frac{1}{8}e$$

c Voor f geldt $y = \frac{\ln(x) - 1}{2 \ln(x) - 3}$, dus voor f^{inv} geldt $x = \frac{\ln(y) - 1}{2 \ln(y) - 3}$

$$2x \ln(y) - 3x = \ln(y) - 1$$

$$2x \ln(y) - \ln(y) = 3x - 1$$

$$(2x - 1) \ln(y) = 3x - 1$$

$$\ln(y) = \frac{3x - 1}{2x - 1}$$

$$y = e^{\frac{3x-1}{2x-1}}$$

Dus $f^{\text{inv}}(x) = e^{\frac{3x-1}{2x-1}}$.

$f^{\text{inv}}(x) = g(x)$ geeft $e^{\frac{3x-1}{2x-1}} = e^{2x}$

$$\frac{3x-1}{2x-1} = 2x$$

$$2x(2x-1) = 3x-1$$

$$4x^2 - 2x = 3x-1$$

$$4x^2 - 5x + 1 = 0$$

$$(4x-1)(x-1) = 0$$

$$x = \frac{1}{4} \vee x = 1$$

$g(\frac{1}{4}) = e^{\frac{1}{2}} = \sqrt{e}$ en $g(1) = e^2$

Dus $E(\frac{1}{4}, \sqrt{e})$ en $F(1, e^2)$.

61 a $\lim_{x \uparrow 0} f(x) = \lim_{x \uparrow 0} \frac{\ln(x^2) - 4}{\ln(x^2) - 1} = \lim_{x \uparrow 0} \frac{2 \ln|x| - 4}{2 \ln|x| - 1} = \lim_{\ln|x| \rightarrow -\infty} \frac{2 \ln|x| - 4}{2 \ln|x| - 1} = \lim_{\ln|x| \rightarrow -\infty} \frac{2 - \frac{4}{\ln|x|}}{2 - \frac{1}{\ln|x|}} = \frac{2-0}{2-0} = 1$

$\lim_{x \downarrow 0} f(x) = \lim_{x \downarrow 0} \frac{\ln(x^2) - 4}{\ln(x^2) - 1} = \lim_{x \downarrow 0} \frac{2 \ln(x) - 4}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow -\infty} \frac{2 \ln(x) - 4}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow -\infty} \frac{2 - \frac{4}{\ln(x)}}{2 - \frac{1}{\ln(x)}} = \frac{2-0}{2-0} = 1$

b $\ln(x^2) - 1 = 0 \wedge \ln(x^2) - 4 \neq 0$

$\ln(x^2) = 1 \wedge \ln(x^2) \neq 4$

$x^2 = e$

$x = \sqrt{e} \vee x = -\sqrt{e}$

De verticale asymptoten zijn de lijnen $x = \sqrt{e}$ en $x = -\sqrt{e}$.

$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{\ln(x^2) - 4}{\ln(x^2) - 1} = \lim_{x \rightarrow \infty} \frac{2 \ln(x) - 4}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow \infty} \frac{2 \ln(x) - 4}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow \infty} \frac{2 - \frac{4}{\ln(x)}}{2 - \frac{1}{\ln(x)}} = \frac{2-0}{2-0} = 1$

De horizontale asymptoot is de lijn $y = 1$.

c $f(x) = 10$ geeft $\frac{\ln(x^2) - 4}{\ln(x^2) - 1} = 10$

$$10 \ln(x^2) - 10 = \ln(x^2) - 4$$

$$9 \ln(x^2) = 6$$

$$\ln(x^2) = \frac{2}{3}$$

$$x^2 = e^{\frac{2}{3}}$$

$$x = e^{\frac{1}{3}} \vee x = -e^{\frac{1}{3}}$$

$$x = \sqrt[3]{e} \vee x = -\sqrt[3]{e}$$

$f(x) \leq 10$ geeft $x < -\sqrt[3]{e} \vee -\sqrt[3]{e} \leq x < 0 \vee 0 < x \leq \sqrt[3]{e} \vee x > \sqrt[3]{e}$

$$\begin{aligned} \text{d } f(x) = 0 \text{ geeft } \ln(x^2) - 4 &= 0 \\ \ln(x^2) &= 4 \\ x^2 &= e^4 \\ x &= e^2 \vee x = -e^2 \end{aligned}$$

Dus $A(e^2, 0)$.

$$f(x) = \frac{\ln(x^2) - 4}{\ln(x^2) - 1} \text{ geeft}$$

$$f'(x) = \frac{(\ln(x^2) - 1) \cdot \frac{2}{x} - (\ln(x^2) - 4) \cdot \frac{2}{x}}{(\ln(x^2) - 1)^2} = \frac{2 \ln(x^2) - 2 - 2 \ln(x^2) + 8}{x(\ln(x^2) - 1)^2} = \frac{6}{x(\ln(x^2) - 1)^2}$$

$$rc_k = f'(e^2) = \frac{6}{e^2(\ln(e^4) - 1)^2} = \frac{6}{e^2(4 - 1)^2} = \frac{6}{9e^2} = \frac{2}{3e^2}$$

$$k \perp l, \text{ dus } rc_k \cdot rc_l = -1$$

$$\frac{2}{3e^2} \cdot rc_l = -1$$

$$rc_l = -1\frac{1}{2}e^2$$

$$\begin{aligned} y &= -1\frac{1}{2}e^2x + b \\ \text{door } A(e^2, 0) \quad &\left. \begin{aligned} -1\frac{1}{2}e^2 \cdot e^2 + b &= 0 \\ b &= 1\frac{1}{2}e^4 \end{aligned} \right\} \end{aligned}$$

$$\text{Dus } l: y = -1\frac{1}{2}e^2x + 1\frac{1}{2}e^4 \text{ en } B(0, 1\frac{1}{2}e^4).$$

$$62 \quad f(x) = \frac{\ln^2(x) - 1}{\ln(x) - 1} = \frac{(\ln(x) - 1)(\ln(x) + 1)}{\ln(x) - 1}$$

Voor de perforatie geldt $\ln(x) - 1 = 0$

$$\ln(x) = 1$$

$$x = e$$

$$\lim_{x \rightarrow e} f(x) = \lim_{x \rightarrow e} \frac{(\ln(x) - 1)(\ln(x) + 1)}{\ln(x) - 1} = \lim_{x \rightarrow e} (\ln(x) + 1) = \ln(e) + 1 = 1 + 1 = 2$$

De perforatie is $A(e, 2)$.

$$g(x) = \ln(x) + 1 \text{ geeft } g'(x) = \frac{1}{x}$$

$$g'(e) = \frac{1}{e}$$

$$\frac{1}{e} \cdot rc_k = -1 \text{ geeft } rc_k = -e$$

$$\begin{aligned} y &= -ex + b \\ \text{door } A(e, 2) \quad &\left. \begin{aligned} -e \cdot e + b &= 2 \\ b &= e^2 + 2 \end{aligned} \right\} \end{aligned}$$

$$\text{Dus } k: y = -ex + e^2 + 2.$$

$$k \text{ snijden met de } x\text{-as geeft } -ex + e^2 + 2 = 0$$

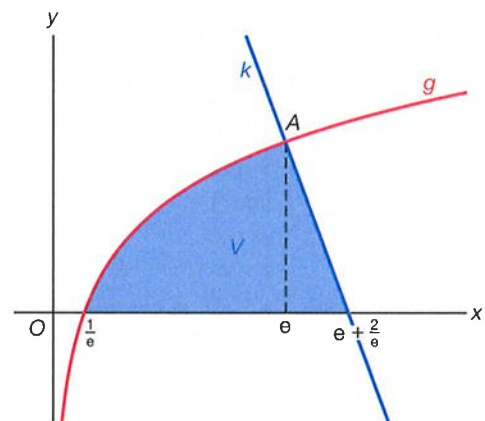
$$ex = e^2 + 2$$

$$x = e + \frac{2}{e}$$

$$g(x) = 0 \text{ geeft } \ln(x) + 1 = 0$$

$$\ln(x) = -1$$

$$x = e^{-1} = \frac{1}{e}$$



$$\begin{aligned} O(V) &= \int_{\frac{1}{e}}^e (\ln(x) + 1) dx + O(\text{driehoek}) = [x \ln(x) - x + x]_{\frac{1}{e}}^e + \frac{1}{2} \cdot \frac{2}{e} \cdot (\ln(e) + 1) = [x \ln(x)]_{\frac{1}{e}}^e + \frac{1}{2} \cdot \frac{2}{e} \cdot 2 \\ &= e \ln(e) - \frac{1}{e} \ln\left(\frac{1}{e}\right) + \frac{2}{e} = e + \frac{1}{e} + \frac{2}{e} = e + \frac{3}{e} \end{aligned}$$

Diagnostische toets

Bladzijde 42

1 a $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3} = \lim_{x \rightarrow -3} \frac{(x + 3)(x - 3)}{x + 3} = \lim_{x \rightarrow -3} (x - 3) = -6$

b $\lim_{x \rightarrow -3} \frac{x^2 + 5x + 6}{x^2 + x - 6} = \lim_{x \rightarrow -3} \frac{(x + 2)(x + 3)}{(x - 2)(x + 3)} = \lim_{x \rightarrow -3} \frac{x + 2}{x - 2} = \frac{-1}{-5} = \frac{1}{5}$

2 a $f_a(x) = \frac{4x^2 + 2x - 12}{2x + a} = \frac{(2x + 4)(2x - 3)}{2x + a}$

Er is een perforatie als $a = 4$ en als $a = -3$.

$\lim_{x \rightarrow -2} f_4(x) = \lim_{x \rightarrow -2} \frac{(2x + 4)(2x - 3)}{2x + 4} = \lim_{x \rightarrow -2} (2x - 3) = -4 - 3 = -7$

Voor $a = 4$ is de perforatie $(-2, -7)$.

$\lim_{x \rightarrow 1\frac{1}{2}} f_{-3}(x) = \lim_{x \rightarrow 1\frac{1}{2}} \frac{(2x + 4)(2x - 3)}{2x - 3} = \lim_{x \rightarrow 1\frac{1}{2}} (2x + 4) = 3 + 4 = 7$

Voor $a = -3$ is de perforatie $(1\frac{1}{2}, 7)$.

b $f_a(x) = \frac{2x^2 - x - 6}{2x + a} = \frac{(x - 2)(2x + 3)}{2x + a} = \frac{\frac{1}{2}(2x - 4)(2x + 3)}{2x + a}$

Er is een perforatie als $a = -4$ en als $a = 3$.

$\lim_{x \rightarrow 2} f_{-4}(x) = \lim_{x \rightarrow 2} \frac{\frac{1}{2}(2x - 4)(2x + 3)}{2x - 4} = \lim_{x \rightarrow 2} (\frac{1}{2}(2x + 3)) = \frac{1}{2}(4 + 3) = 3\frac{1}{2}$

Voor $a = -4$ is de perforatie $(2, 3\frac{1}{2})$.

$\lim_{x \rightarrow -1\frac{1}{2}} f_3(x) = \lim_{x \rightarrow -1\frac{1}{2}} \frac{\frac{1}{2}(2x - 4)(2x + 3)}{2x + 3} = \lim_{x \rightarrow -1\frac{1}{2}} (\frac{1}{2}(2x - 4)) = \frac{1}{2}(-3 - 4) = -3\frac{1}{2}$

Voor $a = 3$ is de perforatie $(-1\frac{1}{2}, -3\frac{1}{2})$.

3 a $\lim_{x \uparrow 1} f_p(x) = \lim_{x \uparrow 1} 3^{x-p} = 3^{1-p}$

$\lim_{x \downarrow 1} f_p(x) = \lim_{x \downarrow 1} (x^2 + 8) = 1^2 + 8 = 9$

$\lim_{x \rightarrow 1} f_p(x)$ bestaat als $3^{1-p} = 9$

$$3^{1-p} = 3^2$$

$$1 - p = 2$$

$$p = -1$$

b $\lim_{x \uparrow 1} f_p(x) = \lim_{x \uparrow 1} |px - 3| = |p - 3|$

$\lim_{x \downarrow 1} f_p(x) = \lim_{x \downarrow 1} (x^2 + 3x) = 1^2 + 3 \cdot 1 = 4$

$\lim_{x \rightarrow 1} f_p(x)$ bestaat als $|p - 3| = 4$

$$p - 3 = 4 \vee p - 3 = -4$$

$$p = 7 \vee p = -1$$

4 a $f_p'(x) = (2x-2)(x+p) = 2x^2 + 2px - 2x - 2p$ voor $x+p \geq 0$, dus $x \geq -p$
 $f_p'(x) = (2x-2)(-x-p) = -2x^2 - 2px + 2x + 2p$ voor $x < -p$
 $\lim_{x \uparrow -p} f_p'(x) = \lim_{x \uparrow -p} (-4x - 2p + 2) = 4p - 2p + 2 = 2p + 2$
 $\lim_{x \downarrow -p} f_p'(x) = \lim_{x \downarrow -p} (4x + 2p - 2) = -4p + 2p - 2 = -2p - 2$
 Geen knik als $\lim_{x \uparrow -p} f_p'(x) = \lim_{x \downarrow -p} f_p'(x)$, dus als $2p + 2 = -2p - 2$
 $4p = -4$
 $p = -1$

b $f_4(x) = \begin{cases} 2x^2 + 8x - 2x - 8 = 2x^2 + 6x - 8 & \text{voor } x \geq -4 \\ -2x^2 - 8x + 2x + 8 = -2x^2 - 6x + 8 & \text{voor } x < -4 \end{cases}$
 $\lim_{x \uparrow -4} f_4'(x) = \lim_{x \uparrow -4} (-4x - 6) = 16 - 6 = 10$, dus $rc_k = 10$.
 $rc_k = \tan(\alpha) = 10$, dus $\alpha = 84,2...^\circ$.
 $\lim_{x \downarrow -4} f_4'(x) = \lim_{x \downarrow -4} (4x + 6) = -16 + 6 = -10$, dus $rc_l = -10$.
 $rc_l = \tan(\beta) = -10$, dus $\beta = -84,2...^\circ$.
 $\alpha - \beta = 84,2...^\circ - (-84,2...^\circ) \approx 169^\circ$
 Dus $\angle(k, l) \approx 180^\circ - 169^\circ = 11^\circ$.

c Zie a.
 $rc_k = 2p + 2$ en $rc_l = -2p - 2$
 $k \perp l$, dus $rc_k \cdot rc_l = -1$
 $(2p + 2)(-2p - 2) = -1$
 $(2p + 2)^2 = 1$
 $2p + 2 = 1 \vee 2p + 2 = -1$
 $2p = -1 \vee 2p = -3$
 $p = -\frac{1}{2} \vee p = -1\frac{1}{2}$

5 a $\lim_{x \rightarrow \infty} \frac{x(5x+2)^2}{x^3+1} = \lim_{x \rightarrow \infty} \frac{x(25x^2+20x+4)}{x^3+1} = \lim_{x \rightarrow \infty} \frac{25x^3+20x^2+4x}{x^3+1} = \lim_{x \rightarrow \infty} \frac{25 + \frac{20}{x} + \frac{4}{x^2}}{1 + \frac{1}{x^3}} = \frac{25+0+0}{1+0} = 25$

b $\lim_{x \rightarrow \infty} \frac{|8-3x^3|}{2x^3+50x} = \lim_{x \rightarrow \infty} \frac{-8+3x^3}{2x^3+50x} = \lim_{x \rightarrow \infty} \frac{-\frac{8}{x^3}+3}{2+\frac{50}{x^2}} = \frac{0+3}{2+0} = 1\frac{1}{2}$

6 a $x^2 - x - 12 = 0 \wedge 2x^2 - 1 \neq 0$
 $(x+3)(x-4) = 0 \wedge 2x^2 \neq 1$
 $x = -3 \vee x = 4$

De verticale asymptoten zijn de lijnen $x = -3$ en $x = 4$.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{2x^2-1}{x^2-x-12} = \lim_{x \rightarrow \infty} \frac{2-\frac{1}{x^2}}{1-\frac{1}{x}-\frac{12}{x^2}} = \frac{2-0}{1-0-0} = 2$$

De horizontale asymptoot is de lijn $y = 2$.

b $f(x) = 2$ geeft $\frac{2x^2-1}{x^2-x-12} = 2$
 $2x^2-1 = 2(x^2-x-12)$
 $2x^2-1 = 2x^2-2x-24$
 $2x = -23$
 $x = -11\frac{1}{2}$

Dus $A(-11\frac{1}{2}, 2)$.

Bladzijde 43

7 teller = 0 geeft $2 \cos(x) - 1 = 0$

$$2 \cos(x) = 1$$

$$\cos(x) = \frac{1}{2}$$

$$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi$$

x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = \frac{2}{3}\pi$

noemer = 0 geeft $8 \sin^2(x) - 6 = 0$

$$8 \sin^2(x) = 6$$

$$\sin^2(x) = \frac{3}{4}$$

$$\sin(x) = \frac{1}{2}\sqrt{3} \vee \sin(x) = -\frac{1}{2}\sqrt{3}$$

$$x = \frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = -\frac{1}{3}\pi + k \cdot 2\pi \vee x = \frac{1}{3}\pi + k \cdot 2\pi$$

x in $[0, 2\pi]$ geeft $x = \frac{1}{3}\pi \vee x = \frac{2}{3}\pi \vee x = \frac{4}{3}\pi \vee x = \frac{5}{3}\pi$

De verticale asymptoten zijn de lijnen $x = \frac{2}{3}\pi$ en $x = \frac{4}{3}\pi$.

$$f(x) = \frac{2 \cos(x) - 1}{8 \sin^2(x) - 6} = \frac{2 \cos(x) - 1}{8(1 - \cos^2(x)) - 6} = \frac{2 \cos(x) - 1}{2 - 8 \cos^2(x)} = \frac{2 \cos(x) - 1}{-2(4 \cos^2(x) - 1)} = \frac{2 \cos(x) - 1}{-2(2 \cos(x) + 1)(2 \cos(x) - 1)}$$

$$\lim_{x \rightarrow \frac{2}{3}\pi} f(x) = \lim_{x \rightarrow \frac{2}{3}\pi} \frac{2 \cos(x) - 1}{-2(2 \cos(x) + 1)(2 \cos(x) - 1)} = \lim_{x \rightarrow \frac{2}{3}\pi} \frac{1}{-2(2 \cos(x) + 1)} = \frac{1}{-2(2 \cdot \frac{1}{2} + 1)} = -\frac{1}{4}$$

$$\lim_{x \rightarrow \frac{4}{3}\pi} f(x) = \lim_{x \rightarrow \frac{4}{3}\pi} \frac{2 \cos(x) - 1}{-2(2 \cos(x) + 1)(2 \cos(x) - 1)} = \lim_{x \rightarrow \frac{4}{3}\pi} \frac{1}{-2(2 \cos(x) + 1)} = \frac{1}{-2(2 \cdot \frac{1}{2} + 1)} = -\frac{1}{4}$$

De perforaties zijn de punten $(\frac{1}{3}\pi, -\frac{1}{4})$ en $(\frac{5}{3}\pi, -\frac{1}{4})$.

8 a $x - 1 = 0 \wedge 3x^2 + 5x - 2 \neq 0$

$$x = 1 \wedge 3x^2 + 5x - 2 \neq 0$$

$$x = 1$$

De verticale asymptoot is de lijn $x = 1$.

$$f(x) = \frac{3x^2 + 5x - 2}{x - 1} = \frac{3x(x - 1) + 3x + 5x - 2}{x - 1} = 3x + \frac{8x - 2}{x - 1} = 3x + \frac{8(x - 1) + 8 - 2}{x - 1} = 3x + 8 + \frac{6}{x - 1}$$

$$\lim_{x \rightarrow \infty} \frac{6}{x - 1} = 0, \text{ dus de scheve asymptoot is de lijn } y = 3x + 8.$$

b $x^2 - 4 = 0 \wedge x^3 + 5x^2 - 4x - 19 \neq 0$

$$x^2 = 4 \wedge x^3 + 5x^2 - 4x - 19 \neq 0$$

$$(x = 2 \vee x = -2) \wedge x^3 + 5x^2 - 4x - 19 \neq 0$$

$$x = 2 \vee x = -2$$

De verticale asymptoten zijn de lijnen $x = 2$ en $x = -2$.

$$g(x) = \frac{x^3 + 5x^2 - 4x - 19}{x^2 - 4} = \frac{x(x^2 - 4) + 4x + 5x^2 - 4x - 19}{x^2 - 4} = x + \frac{5x^2 - 19}{x^2 - 4} = x + \frac{5(x^2 - 4) + 20 - 19}{x^2 - 4} = x + 5 + \frac{1}{x^2 - 4}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x^2 - 4} = 0, \text{ dus de scheve asymptoot is de lijn } y = x + 5.$$

$$9 \quad f(x) = \frac{x^3 + x^2 - 12x}{x^2 - 9} = \frac{x(x^2 + x - 12)}{(x+3)(x-3)} = \frac{x(x+4)(x-3)}{(x+3)(x-3)}$$

$$\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{x(x+4)(x-3)}{(x+3)(x-3)} = \lim_{x \rightarrow 3} \frac{x(x+4)}{x+3} = \frac{3(3+4)}{3+3} = 3\frac{1}{2}$$

Dus de perforatie is $A(3, 3\frac{1}{2})$.

$$(x+3)(x-3) = 0 \wedge x(x+4)(x-3) \neq 0$$

$$(x=3 \vee x=-3) \wedge x \neq 0 \wedge x \neq -4 \wedge x \neq 3$$

$$x = -3$$

Dus de verticale asymptoot is de lijn $x = -3$.

$$f(x) = \frac{x(x+4)}{x+3} = \frac{x^2 + 4x}{x+3} = \frac{x(x+3) - 3x + 4x}{x+3} = x + \frac{x}{x+3} = x + \frac{x+3-3}{x+3} = x + 1 - \frac{3}{x+3}$$

$$\lim_{x \rightarrow \infty} \frac{3}{x+3} = 0, \text{ dus de scheve asymptoot is de lijn } y = x + 1.$$

$$\left. \begin{array}{l} y = x + 1 \\ x = -3 \end{array} \right\} y = -3 + 1 = -2$$

Dus $B(-3, -2)$.

Stel $k: y = ax + b$ met $a = \frac{3\frac{1}{2} - (-2)}{3 - (-3)} = \frac{11}{12}$.

$$\left. \begin{array}{l} y = \frac{11}{12}x + b \\ \text{door } B(-3, -2) \end{array} \right\} \begin{array}{l} \frac{11}{12} \cdot (-3) + b = -2 \\ -2\frac{3}{4} + b = -2 \\ b = \frac{3}{4} \end{array}$$

Dus $k: y = \frac{11}{12}x + \frac{3}{4}$ en het snijpunt met de y -as is $C(0, \frac{3}{4})$.

$$10 \quad a \quad \lim_{x \rightarrow \infty} (10 - 20 \cdot (\frac{9}{10})^x) = 10 - 20 \cdot 0 = 10$$

$$b \quad \lim_{x \rightarrow -\infty} \frac{e^x - 5}{e^x + 20} = \frac{0 - 5}{0 + 20} = -\frac{1}{4}$$

$$c \quad \lim_{x \rightarrow \infty} \frac{e^{x+2} + 1}{e^{2x} + 1} = \lim_{x \rightarrow \infty} \frac{e^2 \cdot e^x + 1}{e^{2x} + 1} = \lim_{x \rightarrow \infty} \frac{\frac{e^2}{e^x} + \frac{1}{e^{2x}}}{1 + \frac{1}{e^{2x}}} = \frac{0 + 0}{1 + 0} = 0$$

$$d \quad \lim_{x \rightarrow -\infty} \frac{5 \cdot 3^x + 2}{4 \cdot 2^x + 1} = \frac{5 \cdot 0 + 2}{4 \cdot 0 + 1} = 2$$

$$11 \quad a \quad f_{3,4}(x) = \frac{3e^x}{2e^x - 4e}$$

$$2e^x - 4e = 0 \wedge 3e^x \neq 0$$

$$2e^x = 4e \wedge e^x \neq 0$$

$$e^x = 2e \wedge e^x \neq 0$$

$$x = \ln(2e)$$

De verticale asymptoot is de lijn $x = \ln(2e)$.

$$\lim_{x \rightarrow -\infty} f_{3,4}(x) = \lim_{x \rightarrow -\infty} \frac{3e^x}{2e^x - 4e} = \frac{3 \cdot 0}{2 \cdot 0 - 4e} = 0$$

$$\lim_{x \rightarrow \infty} f_{3,4}(x) = \lim_{x \rightarrow \infty} \frac{3e^x}{2e^x - 4e} = \lim_{x \rightarrow \infty} \frac{3}{2 - \frac{4e}{e^x}} = \frac{3}{2 - 0} = 1\frac{1}{2}$$

De horizontale asymptoten zijn de lijnen $y = 0$ en $y = 1\frac{1}{2}$.

b De verticale asymptoot is de lijn $x = 3$, dus er geldt $2e^x - qe = 0$ voor $x = 3$.
Dit geeft $qe = 2e^3$ oftewel $q = 2e^2$.

$$\lim_{x \rightarrow \infty} f_{p,q}(x) = \lim_{x \rightarrow \infty} \frac{pe^x}{2e^x - qe} = \lim_{x \rightarrow \infty} \frac{p}{2 - \frac{qe}{e^x}} = \frac{p}{2 - 0} = \frac{1}{2}p$$

De horizontale asymptoot is de lijn $y = \frac{1}{2}p$.

$$\frac{1}{2}p = 4 \text{ geeft } p = 8$$

Dus voor $p = 8$ en $q = 2e^2$.

$$12 \quad \mathbf{a} \quad \lim_{x \rightarrow \infty} \frac{2 \log(x)}{1 + 16 \log(x)} = \lim_{x \rightarrow \infty} \frac{2 \log(x)}{1 + \frac{2 \log(x)}{2 \log(16)}} = \lim_{x \rightarrow \infty} \frac{2 \log(x)}{1 + \frac{2 \log(x)}{4}} = \lim_{2 \log(x) \rightarrow \infty} \frac{2 \log(x)}{1 + \frac{1}{4} \cdot 2 \log(x)} =$$

$$\lim_{2 \log(x) \rightarrow \infty} \frac{1}{\frac{1}{2 \log(x)} + \frac{1}{4}} = \frac{1}{0 + \frac{1}{4}} = 4$$

$$\mathbf{b} \quad \lim_{x \rightarrow \infty} \frac{\ln(x^3)}{1 - 3 \ln(x)} = \lim_{x \rightarrow \infty} \frac{3 \ln(x)}{1 - 3 \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{3 \ln(x)}{1 - 3 \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{3}{\frac{1}{\ln(x)} - 3} = \frac{3}{0 - 3} = -1$$

$$\mathbf{c} \quad \lim_{x \rightarrow \infty} \frac{3 + \log(x)}{5 + \ln(x)} = \lim_{x \rightarrow \infty} \frac{3 + \frac{\ln(x)}{\ln(10)}}{5 + \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{3 + \frac{\ln(x)}{\ln(10)}}{5 + \ln(x)} = \lim_{\ln(x) \rightarrow \infty} \frac{\frac{3}{\ln(x)} + \frac{1}{\ln(10)}}{\frac{5}{\ln(x)} + 1} = \frac{0 + \frac{1}{\ln(10)}}{0 + 1} = \frac{1}{\ln(10)}$$

$$13 \quad \mathbf{a} \quad \lim_{x \downarrow 0} f(x) = \lim_{x \downarrow 0} \frac{4 \ln(x) - 1}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow -\infty} \frac{4 \ln(x) - 1}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow -\infty} \frac{4 - \frac{1}{\ln(x)}}{2 - \frac{1}{\ln(x)}} = \frac{4 - 0}{2 - 0} = 2$$

$$\mathbf{b} \quad 2 \ln(x) - 1 = 0 \wedge 4 \ln(x) - 1 \neq 0$$

$$2 \ln(x) = 1 \wedge 4 \ln(x) \neq 1$$

$$\ln(x) = \frac{1}{2} \wedge \ln(x) \neq \frac{1}{4}$$

$$x = \sqrt{e}$$

De verticale asymptoot is de lijn $x = \sqrt{e}$.

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{4 \ln(x) - 1}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow \infty} \frac{4 \ln(x) - 1}{2 \ln(x) - 1} = \lim_{\ln(x) \rightarrow \infty} \frac{4 - \frac{1}{\ln(x)}}{2 - \frac{1}{\ln(x)}} = \frac{4 - 0}{2 - 0} = 2$$

De horizontale asymptoot is de lijn $y = 2$.